

2004

Threshold cointegration test of the Fisher effect

Biyong Xu
Iowa State University

Follow this and additional works at: <https://lib.dr.iastate.edu/rtd>

 Part of the [Finance Commons](#), and the [Finance and Financial Management Commons](#)

Recommended Citation

Xu, Biyong, "Threshold cointegration test of the Fisher effect " (2004). *Retrospective Theses and Dissertations*. 1207.
<https://lib.dr.iastate.edu/rtd/1207>

This Dissertation is brought to you for free and open access by the Iowa State University Capstones, Theses and Dissertations at Iowa State University Digital Repository. It has been accepted for inclusion in Retrospective Theses and Dissertations by an authorized administrator of Iowa State University Digital Repository. For more information, please contact digirep@iastate.edu.

Threshold cointegration test of the Fisher effect

by

Biyong Xu

A dissertation submitted to the graduate faculty
in partial fulfillment of the requirements for the degree of
DOCTOR OF PHILOSOPHY

Major: Economics

Program of Study Committee:

Barry Falk, Major Professor

Helle Bunzel

Wayne Fuller

Peter Orazem

John Schroeter

Iowa State University

Ames, Iowa

2004

Copyright © Biyong Xu, 2004. All rights reserved.

UMI Number: 3158382

INFORMATION TO USERS

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleed-through, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

UMI[®]

UMI Microform 3158382

Copyright 2005 by ProQuest Information and Learning Company.

All rights reserved. This microform edition is protected against unauthorized copying under Title 17, United States Code.

ProQuest Information and Learning Company
300 North Zeeb Road
P.O. Box 1346
Ann Arbor, MI 48106-1346

Graduate College
Iowa State University

This is to certify that the doctoral dissertation of

Biyong Xu

has met the dissertation requirements of Iowa State University

Signature was redacted for privacy.

Major Professor

Signature was redacted for privacy.

For the Major Program

To my mother and to the memory of my father

TABLE OF CONTENTS

INTRODUCTION.....	1
CHAPTER ONE: LITERATURE REVIEW ON THE FISHER EFFECT.....	5
1.1 The Fisher Hypothesis	5
1.2 Literature Review.....	9
1.3 Threshold Cointegration Test of the Fisher Effect.....	17
1.4 Description of Data	34
CHAPTER TWO: UNIT ROOT TESTS.....	37
2.1 Linear Unit Root Tests.....	37
2.2 Unit Root Test under the Threshold Alternative.....	42
2.3 End-of-chapter Summary	45
CHAPTER THREE: COINTEGRATION TESTS	46
3.1 Linear Cointegration Tests.....	46
3.2 Nonlinearity Tests	53
3.3 End-of-chapter Summary.....	56
CHAPTER FOUR: THRESHOLD ERROR CORRECTION MODEL.....	57
4.1 Hansen and Seo's (2002) Two-Regime TVECM	57
4.2 The Estimated Threshold Error Correction Model	62
4.3 Test of Equal Forecast Efficiency.....	67
4.4 End-of-Chapter Summary	73
CHAPTER FIVE: CONCLUSIONS AND FINAL REMARKS	74
APPENDIX A: TECHNICAL BACKGROUND	77
APPENDIX B: TABLES	80
APPENDIX C: FIGURES.....	86
REFERENCES.....	88
ACKNOWLEDGEMENTS	92

INTRODUCTION

Interest rate and inflation are two fundamental variables in the economy. For decades, economists have been trying to disclose the relationship between them. One of the most well-known hypotheses is the Fisher hypothesis, which was first proposed by the famous economist Irving Fisher in Fisher (1930). According to the hypothesis, the nominal interest rate on bonds moves one-to-one with the rate of inflation anticipated by the public, and the expected real rate of return is constant over time. The hypothesis implies that in the long run there is a one-to-one correspondence between changes of the nominal interest rate and the changes of inflation, which is often referred to as the Fisher effect in the literature.

The Fisher hypothesis, however, is controversial in both macroeconomic theories and empirical studies. Different macroeconomic and financial models give conflicting explanations of the relationship between the nominal interest rate and the inflation rate. For example, we have the hypothesis of superneutrality of money, which claims that the inflation does not affect real variables, but at the same time we also have proposition of Tobin effect, which describes a possible negative relationship between the real interest rate and the inflation that depresses the Fisher effect¹.

The empirical studies do not help much to reduce the controversy in the theoretical literature. Fama (1975) argues that the nominal interest rate is the best possible predictor of

¹ See Section 1.2.1 for more details on the theoretical literature.

the inflation rate and claims that the Fisher effect holds in the United States. With ADF and Philips-Perron tests, Ross (1988) claims that US ex post real interest rate is nonstationary, which can be a contradiction to the assumption of constant expected real interest rate inherent in the Fisher hypothesis. By applying Engle and Granger (1987) cointegration procedure, Mishkin (1992) asserts that the nominal interest rate and the inflation rate are cointegrated and that the Fisher effect exists in the long run. With the inflation rate modeled by a Markov regime switching process, Evans and Lewis (1995) argue that the rational anticipation of infrequent shifts in the inflation process could have led to a significant downward bias in the estimate of the long-run Fisher effect. Crowder and Hoffman (1996) consider the tax-adjusted Fisher equation in Johansen (1988) cointegration framework and claim that the estimated Fisher effect is consistent with the theoretically predicted value.¹

Most of the previous empirical studies are using linear models in time series, which was predicated on the assumption that the path of adjustment towards long-run equilibrium is necessarily symmetric. The assumption of symmetric adjustment, however, may not be warranted. It is frequently argued that that some fundamental economic variables, including the real GNP and the unemployment rate, display asymmetric adjustment paths, which cannot be properly modeled by linear models². Since the real interest rate is closely related to these variables, it may also follow an asymmetric adjustment path.

¹ See Section 1.2.2 for more details on the empirical literature.

² See Neftci (1984) and Hamilton (1989) for examples.

In this dissertation, we are going to study the Fisher relationship within a fresh nonlinear framework. The dissertation is filling several blanks in the empirical literature.

1. *Testing the stationarity of the nominal interest rate and the inflation rate under a nonlinear threshold autoregressive model (TAR).* If the nominal interest rate and/or the inflation rate follow a TAR process, linear unit root tests¹ are misspecified under the alternative and therefore their power will suffer. To address the possible power distortion, Enders and Granger (1998) test, which allows an asymmetric path of adjustment, will be performed to check the order of integration.
2. *Testing threshold cointegration between the nominal interest rate and the inflation rate.* To test for possible nonlinearity in the Fisher relationship, threshold cointegration analysis described in Balke and Fomby (1997) is to be implemented.
3. *Modeling the Fisher relationship in a TVECM framework.* A two-regime threshold vector error correction model (TVECM) described in Hansen and Seo (2002) will be applied to capture the nonlinearity in the Fisher relationship. Further more, the encompassing tests described in Clark and McCracken (2001) will be carried out to compare the performance of the linear cointegration analysis and the TVECM.

¹ Here the linear unit root tests refer to the class of unit root tests that are linear under both the null and the alternative. For example, the ADF test and Phillips-Perron unit root test.

Correspondingly, there are five chapters in this dissertation. In Chapter One, we will extensively review the literature on the Fisher effect, the threshold cointegration and the TVECM. The results of unit root tests, including one nonlinear unit root test (Enders and Granger (1998)) and two linear unit root tests (ADF and ADF^{GLS}), will be presented in Chapter Two. In Chapter Three, we are going to test for the presence of nonlinearity in the relationship between the nominal interest rate and the inflation rate, following the two-step procedure of the Balke and Fomby (1997). Linear cointegration analyses, including the Johansen (1988) and Phillips-Ouliaris (1990), will be performed in the first step and nonlinearity tests will be applied to the cointegration residuals in the second step. In Chapter Four, we will model the nonlinearity in the Fisher relationship with a two-regime TVECM in Hansen and Seo (2002) and compare its out-of-sample forecast efficiency with the linear cointegration analysis. Chapter Five is the conclusions and directions for future research.

CHAPTER ONE: LITERATURE REVIEW ON THE FISHER EFFECT

1.1 The Fisher Hypothesis

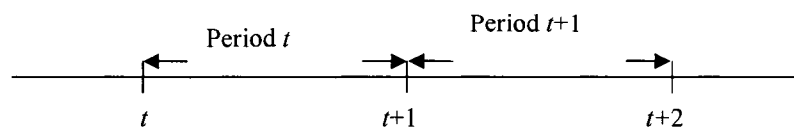
An interesting topic in macroeconomics and financial economics is the relationship between the nominal interest rate and inflation. A well-known hypothesis about this relationship was proposed by Fisher (1930). The original proposition is that the nominal interest rate on bonds is the sum of the expected real interest, which is the expected rate of returns associated with holding real assets, and the rate of inflation anticipated by the public.

1.1.1 The Basic Fisher Equation

The Fisher hypothesis can be summarized in the following mathematical terms

$$i_t = E_t \pi_t + E_t r_t \quad (1.1)$$

where i_t = nominal interest rate at time t for one-period bonds maturing at time $t+1$; $E_t r_t$ = real expected rate of return in period t (the period between time t and $t+1$)¹, which is the expected rate of returns associated with holding real assets; $E_t \pi_t$ = expected inflation rate between time t and $t+1$.



¹ In this dissertation, period t is the time interval between time t and $t+1$. $E_t \pi_t$ and $E_t r_t$ are formed at time t . See the illustration.

The Fisher hypothesis represents one of the oldest and most basic equilibrium relationships in financial economics. The hypothesis is based on the assumption that investors consider assets with yields in real terms, such as equities and physical capital, as very close substitutes for bonds, a class of assets whose returns are in nominal terms. Investor's insistence that such assets bear equivalent real rate of returns enforces (1.1). This part of the Fisher hypothesis, which asserts that the spread between the nominal rate of returns on bonds and the rate of return on real assets fully adjusts to reflect changes in the anticipated rate of inflation, is widely accepted in economics.

Fisher and his followers further assumed that the expected real rate of return, $E_t r_t$, is unaffected by changes in the anticipated rate of inflation. Typically, it has been specified that

$$r_t = \mu + \varepsilon_t$$

where μ is a constant and ε_t is a mean-zero stochastic disturbance that is uncorrelated with the information at the beginning of period t . So $E_t r_t = \mu$ and (1.1) becomes

$$i_t = \mu + E_t \pi_t. \quad (1.2)$$

Here μ is the long-run equilibrium "real" rate of interest, which is presumably determined by the classical factors of productivity and thrift. Equation (1.2) asserts that the ex ante real interest rate, $i_t - E_t \pi_t$, remains unchanged across periods. This part of the hypothesis has been one of the most debated issues in economics, both theoretically and empirically.

To make (1.2) testable, the practice is to assume that the inflation forecast is unbiased so $\pi_t = E_t \pi_t + \eta_t$, where η_t is the forecast residual which is uncorrelated with all information at

the start of period t . Equation (1.2) can be rewritten into either of the following two equations:

$$i_t = \mu + \pi_t - \eta_t \quad (1.3)$$

$$\pi_t = -\mu + i_t + \eta_t \quad (1.4)$$

Although (1.3) and (1.4) are equivalent mathematically, they are different statistically. In particular, $cov(i_t, \eta_t) = 0$ and $cov(\pi_t, \eta_t) \neq 0$. Thus, (1.4) is a valid regression equation, but (1.3) is not.

1.1.2 Extensions of the Basic Fisher Equation

An important extension of the Fisher equation is adding a tax effect to the yield of bonds. Assume there is a tax on the bond yield but no tax on the real yield, then (1.1) becomes

$$(1 - \tau_t) i_t = E_t \pi_t + E_t r_t \quad (1.5)$$

where τ_t is the tax rate on interest in period t . Here we suppose the agents in the economy know the tax rate in period t at the beginning of that period. The left side of (1.5) is the after-tax yield of bonds. Following similar procedures as before we get

$$(1 - \tau_t) i_t = \mu + \pi_t - \eta_t \quad (1.6)$$

Comparing (1.6) with (1.3), we can see the major difference is that the nominal interest rate in (1.6) is tax-adjusted.

There are also extensions of the Fisher hypotheses that incorporate the effect of a risk premium, because the nominal interest rate is less stable than the real interest rate. We choose not to consider the effect of a risk premium because of the following reasons:

1. We need to assume special functional forms, e.g., some utility function and production function, to address the risk premium, which will make our study less general.
2. According to previous studies, the risk premium is relatively small¹.
3. If the risk premium varies randomly around a constant, it can be absorbed in the constant term μ and the error term.

So far our horizon is one-period ahead. The Fisher equation can also be extended to explain the m -period-ahead relationship between the nominal interest rate for bonds and the expected inflation:

$$i_t^m = E_t \pi_t^m + E_t r_t^m \quad (1.7)$$

where i_t^m = the nominal interest rate for bonds maturing at time $t+m$; $E_t r_t^m$ the expected real rate of return between time t and $t+m$; and $E_t \pi_t^m$ = expected inflation rate in the period between time t and $t+m$ (keep in mind that $E_t r_t^m$ and $E_t \pi_t^m$ are formed at time t). Note that if we set $m=1$, (1.7) is reduced to the one-period relationship described in (1.1). Similarly, we can get

$$i_t^m = \mu^m + \pi_t^m - \eta_t^m \quad (1.8)$$

and

¹ See Crowder and Hoffman (1996).

$$(1 - \tau_t^m) i_t^m = \mu^m + \pi_t^m - \eta_t^m \quad (1.9)$$

which are analogous to (1.3) and (1.6), respectively.

The Fisher hypothesis is one of the fundamental assumptions in economics. It has been used in many important models in macroeconomics and financial economics and it is closely related to the idea of superneutrality of money, which asserts that a permanent change in inflation has no long-run effect on the real economic variables, such as unemployment and the real interest rate. At the same time, the Fisher hypothesis has important implications for the behavior of interest rates, the rationality of people's expectation, and the efficiency and maturity of financial markets.

1.2 Literature Review

In this section we are going to look at the existing literature on the Fisher hypothesis, including the theoretical literature and empirical findings.

1.2.1 Theoretical Literature

Different economic and financial models give different and, sometimes, conflicting explanations for the relationship between the nominal interest rate and the inflation. Ahmed and Rogers (1999) summarize the role of the Fisher equation in different macroeconomic models. They consider a general setting in which the economy is represented by an infinitely

lived representative consumer, who is trying to maximize the integrated lifetime loglinear utility function subject to a budget constraint and a cash in advance (CIA) constraint. The production technology uses labor and physical capital (the only good in the society) as inputs. The total time of the representative consumer is divided between leisure and working. The money supply is controlled by the government and is assumed to be exogenous. This is a fairly general setting, and it includes several important special cases.

Model 1: Sidrauski Model (1967). In this model, money enters the utility function, but there is no CIA constraint. The real sector of the economy is not affected by changes in inflation. This is the well-known superneutrality of money. The Fisher equation holds in this model.

Model 2: CIA-for-consumption model. In this model, money provides no direct utility, but cash is needed in advance to finance the consumption expenditure. The model is proposed by Cooley and Hansen (1989). There exists a Fisher effect in this model.

Model 3: CIA-for-consumption-and-investment model. In this model, money is not allowed to enter the utility function but the CIA constraint applies to both consumption and investment. Stockman (1981) and Abel (1985) examine this type of model. The Fisher effect does not exist in this model.

Model 4: Tobin Model. Tobin (1965) argues that agents shift out of nominal assets into real assets in response to an increase in the expected inflation rate. This causes the price of

nominal assets to fall, thus increasing the expected return on them, and it causes the price of real assets to rise, thus reducing their expected returns. This well-known “Tobin effect” results in a negative relationship between inflation and the real rate of interest, thus depressing the Fisher effect.

There are other types of models. Darby (1975) and Feldstein (1976) demonstrate that the taxation of interest implies more-than-complete adjustment of nominal interest to expected inflation. Fama and Gibbons (1982) argue that higher real interest rates result from greater productivity in the economy. The increase in output pushes up money demand. If the increase in money demand is not accompanied by a higher money supply, then those output shocks will push down inflation. Thus, there is a negative correlation between inflation and the real interest rate, which depresses the Fisher effect.

1.2.2 Empirical Literature

One might hope that the controversies in theory about the Fisher hypothesis can be resolved by empirical studies. However, it is no less controversial in the empirical literature. This subsection will introduce previous empirical tests of the Fisher equation. Some basic terminologies involved, such as stationarity, cointegration, spurious regression and error correction model (ECM), are presented in Appendix A.

Fama (1975) investigates the relationship between the Fisher equation and the efficiency of the Treasury bill market. He argues that if the market is efficient, the agents’

expected inflation rate is unbiased for the true value. Moreover, once i_t is set at time t , the details of the information (up to time t) that an efficient market used to assess the expected inflation rate becomes irrelevant. Therefore, the nominal interest rate observed at time t is the best possible predictor of the inflation rate in period t . Fama first used monthly data from January 1953 through July 1971 to test the null hypothesis that the real rate of return is constant. The null is not rejected. He then estimates the following two equations:

$$\pi_t = \mu_0 + \beta_1 i_t + \varepsilon_t, \quad (1.10)$$

$$\pi_t = \mu_0 + \beta_1 i_t + \beta_2 \pi_{t-1} + \varepsilon_t \quad (1.11)$$

The estimate of β_2 in (1.11) is not significantly different from zero, which implies that the information in π_{t-1} is fully utilized in setting the nominal interest i_t . The estimates for β_1 in both (1.10) and (1.11) are not significantly different from one, which implies the Fisher equation holds. The results are extended to Treasury bills with longer maturities. Based on this empirical evidence, Fama comes to the conclusion that the bond market is efficient, the Fisher effect exists and the nominal yield on bonds has predictive content for the inflation in the future. Fama also suggests that previous rejections of the Fisher equation in empirical literature could be the consequence of poor price indices.

Fama (1975) is important yet controversial. Some researchers point out that Fama's sample is extremely unrepresentative of the twentieth century and it contains little variation in the variables of interest. What's more, Fama has not explicitly tested the order of integration of the data he used.

The Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests were used by Rose (1988) to check the stationarity of inflation and the nominal interest rate. Based on data of different frequency, samples, and transformations, Rose claims that the ex post real interest rate is nonstationary and that the OLS estimator from equation (1.10) and (1.11), used by Fama (1975), may suffer spurious regression bias. Consequently, any inferences based on (1.10) and (1.11) may be unreliable. (see Appendix A for the definition of spurious regression.)

Mishkin (1992) points out that the relationship between the short-term interest rate and the inflation rate discussed in Fama (1975) is not robust to the sample chosen. Although the Fisher equation is widely accepted for the period after the Fed-Treasury Accord in 1951 until October 1979 in the United States, it is generally rejected by the data before World War II and after October 1979. To explore the Fisher effect, Mishkin estimated the following regression equation:

$$\pi_t^m = \mu^m + \beta_m i_t^m + \varepsilon_t^m \quad (1.12)$$

where π_t^m = m -period future inflation rate from time t to $t+m$; i_t^m = m -period interest rate known at time t ; $m=1, 3$. Based on the ADF and PP unit root tests, Mishkin concludes that both the inflation and nominal interest rates between January 1953 and December 1990 contain a unit root. However, the unit root hypothesis is rejected for $\pi_t^m - i_t^m$. In addition, the application of the Engle-Granger (1987) procedure suggests the π_t^m and i_t^m are cointegrated. Therefore, the evidence supports the existence of a long-run Fisher effect. The long-run Fisher effect means that when the nominal interest rate is high for a long period of time, the expected inflation rate tends to be high. A short-run Fisher effect, however, means that a

change in the nominal interest rate is followed immediately by a change in expected inflation:

$$\Delta\pi_t^m = \alpha_m + \beta_m \Delta i_t^m + \eta_t^m. \quad (1.13)$$

To address possible correlation between Δi_t^m and η_t^m , Mishkin used a two-step two-stage least square procedure to estimate (1.13). Over the whole sample period, β_m is not significantly different from zero. Therefore, there seems to be no short-run Fisher effect, according to Mishkin's finding.

Evans and Lewis (1995) find that the nominal interest rate and the inflation rate are both $I(1)$ by ADF test and they are cointegrated by the Johansen (1991) and Johansen and Juselius (1990) tests, based on monthly U.S. data from January 1947 to February 1987. To obtain parameter and standard error estimates that correct for the problem of finite sample bias present in the cointegrating equations, they apply the dynamic OLS (DOLS) method developed by Stock and Watson (1993):

$$i_t = \alpha + \beta\pi_t + \sum_{j=-6}^6 \alpha_j \Delta i_{t-j} + v_t \quad (1.14)$$

The null hypothesis $\beta=1$ is strongly rejected so the ex post real interest rate is nonstationary, which is consistent with the findings of Rose (1988). A common interpretation based on a nonstationary ex post real interest rate is that the ex ante real interest rate is also nonstationary, which results in a rejection of the Fisher effect¹. However, Evans and Lewis argue that the ex ante real interest rate can be stationary even if the ex post real interest rate is $I(1)$. To back up their argument, they model the US post-war inflation with a Markov

¹ According to the Fisher hypothesis, the expected real interest rate is constant across time. See Section 1.1.1

regime-switching process. They claim that the rational anticipation of infrequent shifts in the inflation process have led to significant biases in the estimates of the long-run Fisher effect, and it is these small sample biases that create the false appearance of permanent shocks to ex ante real rates even when none are truly present.

Crowder and Hoffman (1996) consider the tax-adjusted “observable” Fisher equation

$$(1-\tau_t)i_t = \mu + \beta\pi_t + \varepsilon_t \quad (1.15)$$

where ε_t is a stationary error process. They apply Johansen’s (1988) procedure to quarterly U.S. data from 1952:Q1 to 1991:Q4. Cointegration is not rejected and the estimated Fisher effect is not significantly different from 1. For comparison, they also apply the same procedure to the data unadjusted for the tax and the results are similar except that the estimated Fisher effect is 1.34, which is significantly different from one. Their estimates of the Fisher effect are consistent with the theoretically predicted value, considering the effect of the interest tax.

Crowder and Hoffman (1996) also use Monte Carlo experiments to compare the efficiencies of the three commonly used procedures in estimating the Fisher equation: the maximum likelihood procedure by Johansen (1988), the two-step OLS procedure by Engle and Granger (1987) and the dynamic OLS (DOLS) method in Stock and Watson (1993). In their simulations, inflation is modeled as an ARIMA(0,1,1) process and the nominal interest rate as an ARIMA(1,1,0) process. The parameterizations of “quarterly” and “monthly” data are based on data of their own and the data used by Evans and Lewis (1995), respectively.

Crowder and Hoffman find a considerable downward bias in the (normalized) cointegrating parameter estimates from the two-step OLS procedure and the DOLS procedure in all experiments. This downward bias occurs in as many as 95% of the repetitions without simulating “breaks” in the dynamic process of inflation, as in Evans and Lewis. At the same time, the application of the Johansen maximum likelihood technique to the monthly data of Mishkin(1992) and Evans and Lewis(1995) yields Fisher effect estimates of 1.35 and 1.36, respectively, which are consistent with the theoretically predicted value, considering tax effect. Therefore, Crowder and Hoffman conclude that the tax-adjusted Fisher equation is valid in the long run.

Malliaropulos (2000) focuses on the effect of possible structural breaks in testing the Fisher equation. As Perron (1989) showed, standard stationarity tests are biased towards nonstationarity since they misinterpret structural breaks as permanent stochastic disturbances. Malliaropulos applies the sequential Augmented Dickey-Fuller tests of Zivot and Andrews (1992), which accounts for structural breaks in the data with endogenous timing, and find strong evidence for the existence of structural breaks in inflation, nominal interest rates and ex post real interest rates. Malliaropulos then estimates the Fisher effect based on the VAR representation in appropriately detrended variables and claims that the Fisher effect exists in the mid-term and long-term.

Some researchers have also studied the Fisher relation outside the United States. For example, Crowder (1997) studies Canadian data following the method proposed by Mishkin

(1992) and Crowder and Hoffman (1996) and concludes that the estimated Fisher effect lies statistically within the range implied by theory.

In summary, there are three “popular” explanations for the possible failure of the Fisher equation in empirical studies. The first explanation is the Tobin effect presented in Section 1.2.1. The second explanation is offered by Evans and Lewis(1995), who hypothesize that regime switches in the sequence of inflation in U.S. may lead to estimates of the Fisher effect that are less than the theoretically implied value. This is the result of the so-called “peso problem”, in which a low probability is attached to a rare event (in our case, high inflation), leading to biased estimators. Finally, a third explanation is that the failure is the result of using inappropriate estimators or misspecified estimation equations.

1.3 Threshold Cointegration Test of the Fisher Effect

Most of the previous empirical studies have used linear models in time series to describe the relationship between inflation and the interest rate, which implies that the path of adjustment towards the long-run equilibrium is necessarily symmetric. To see this, remember that according to previous studies, the nominal interest and inflation rate are cointegrated and therefore there is an error correction representation

$$\begin{pmatrix} \Delta i_t \\ \Delta \pi_t \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} + \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} (i_t - \beta \pi_t) + \varepsilon_t$$

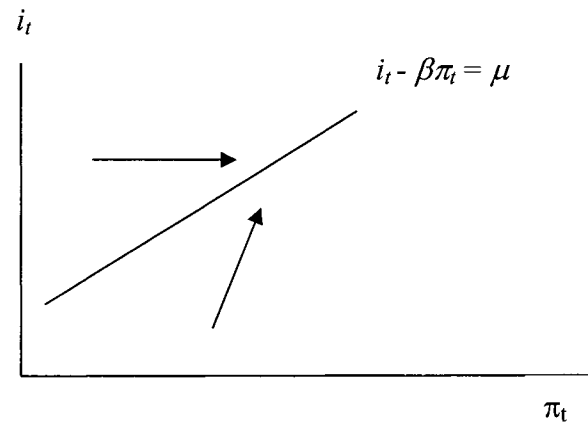


Figure 1: Symmetric Adjustment

In this model, the linear combination $i_t - \beta\pi_t$ is stationary and the adjustment speed $(\theta_1, \theta_2)'$ remain the same whether the system is above or below the equilibrium. The symmetric adjustment model is illustrated in Figure 1.

However, the symmetry assumption may not be warranted. There are two important reasons to look beyond linear models.

1. It is frequently argued that some fundamental economic variables, including GNP and the unemployment rate, display asymmetric adjustment paths (for example, Neftci (1984) and Hamilton (1989)), which cannot be properly modeled by linear models. Since the real interest rate is closely related to these variables, it may also follow a nonlinear path.
2. Normally, the real interest rate should be positive. Therefore, once the nominal interest rate falls below the inflation rate, agents in the economy would expect

the inflation rate to drop and/or the nominal interest rate to rise, which will help the system to drift back to equilibrium. However, this mechanism does not exist when the nominal interest rate is bigger than the inflation rate. So the adjustment to the equilibrium path could be asymmetric.

In the past two decades, non-linear time series models have aroused considerable interest in the field of econometrics. In this dissertation, we are going to look at the relationship between the nominal interest rate and inflation in a fresh nonlinear perspective. Hopefully, this will provide some new insights for the historically controversial Fisher equation.

A useful class of non-linear models is the threshold autoregressive (TAR) model. There exists a rich literature on estimating TAR models. Recent development in econometric methodology enables us to model the Fisher equation in the TAR framework. Since the nominal interest rate and the inflation rate appear to be integrated of order one and cointegrated, as suggested by most of the previous studies, it is appropriate to apply the threshold cointegration model proposed by Balke and Fomby (1997) and further developed by Lo and Zivot (2000). In a model of threshold cointegration,

$$\begin{pmatrix} \Delta i_t \\ \Delta \pi_t \end{pmatrix} = \begin{pmatrix} \alpha_1^{(j)} \\ \alpha_2^{(j)} \end{pmatrix} + \begin{pmatrix} \theta_1^{(j)} \\ \theta_2^{(j)} \end{pmatrix} (i_t - \beta \pi_t) + \varepsilon_t^{(j)} \quad \text{if } \gamma^{(j-1)} \leq i_t - \beta \pi_t \leq \gamma^{(j)}$$

where $j \in \{1, 2, \dots, g\}$ with $\gamma^{(0)} = -\infty$ and $\gamma^{(g)} = +\infty$. In this threshold error correction model, $i_{t-1} - \beta \pi_{t-1}$ is the long-run equilibrium, and $(\theta_1^{(j)}, \theta_2^{(j)})'$ is the regime specific adjustment speed. To get an intuitive understanding of the threshold cointegration model, consider a

two-regime threshold error correction model in which the threshold is the real interest rate μ , as illustrated in Figure 2. That is, $g = 2$ and $\gamma^{(1)} = \mu$. In this example, the speed of adjustment in Regime 1 is faster than in Regime 2, which is quite different from linear models.

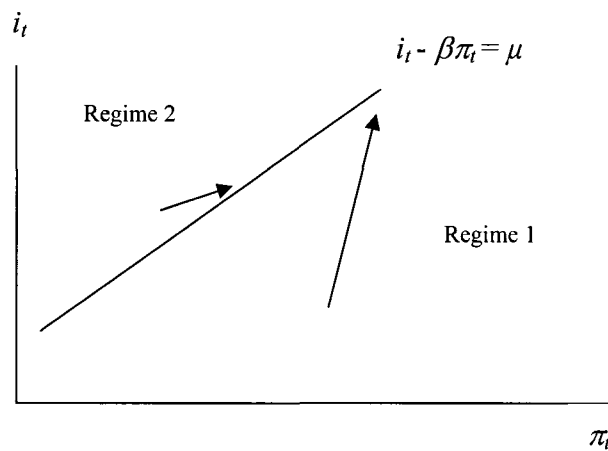


Figure 2: Asymmetric Adjustment

The literature on testing for threshold effects and on threshold cointegration will be presented next.

1.3.1 Literature on Testing for Threshold Effects

A commonly used univariate TAR model is the self-excited TAR model

$$x_t = \beta_0^{(j)} + \beta_1^{(j)}x_{t-1} + \beta_2^{(j)}x_{t-2} + \dots + \beta_p^{(j)}x_{t-p} \text{ if } \gamma^{(j-1)} \leq x_{t-d} \leq \gamma^{(j)} \quad (1.16)$$

where x_t is a univariate stationary time series; p and d are nonnegative integers and $p \geq d$;

$\gamma^j \in \mathfrak{R}$ for $j = \{1, 2, \dots, g\}$ with $\gamma^{(0)} = -\infty$, $\gamma^{(g)} = +\infty$; $\varepsilon_t \sim iid(0, \sigma^2)$ is independent of the past x_{t-1} ,

x_{t-2}, \dots . In this model, $\gamma^{(j)}$ is called the threshold parameter, and d the delay parameter. It is clear that the TAR model is piecewise linear and, as a result, most of the tools developed for linear series can be used with some modifications.

A number of tests have been proposed to test for a threshold effect. Generally, these tests can be classified into two categories: misspecification tests and specification tests. Two commonly used misspecification tests are Petrucci and Davis (1986) and Tsay (1989).

Petrucci and Davis's (1986) method is presented in (1.17), a two-regime TAR model:

$$x_t = \beta_0^{(j)} + \beta_1^{(j)} x_{t-1} + \beta_2^{(j)} x_{t-2} + \dots + \beta_p^{(j)} x_{t-p} + \varepsilon_t^{(j)} \quad \text{if } \gamma^{(j-1)} \leq x_{t-d} \leq \gamma^{(j)} \quad (1.17)$$

where $j \in \{1, 2\}$ with $\gamma^{(0)} = -\infty$, $\gamma^{(2)} = +\infty$. Suppose we want to test the null H_0 under which the

parameters are constant across regimes. Petrucci and Davis suggest using an arranged

regression. Let $x_{(i)}$ be the i^{th} ($i=1, 2, \dots, n-p$) smallest observation among $\{x_{p+1-d}, \dots, x_{n-d}\}$.

Then (1.17) can be formulated as a finite autoregression in the $x_{(i)}$. If the threshold value lies

between the m and $(m+1)^{\text{th}}$ ordered x_t values, the complete p^{th} order autoregression implied

by (1.17) can be rewritten as

$$x_{(i)+d} = \begin{cases} \beta_0^{(1)} + \sum_{l=1}^p \beta_l^{(1)} x_{(i)+d-l} & (i = 1, 2, \dots, m) \\ \beta_0^{(2)} + \sum_{l=1}^p \beta_l^{(2)} x_{(i)+d-l} & (i = m+1, m+2, \dots, n-p) \end{cases} \quad (1.18)$$

If the first s values of the $x_{(i)}$ (for $1 < s_{\min} \leq s \leq m$) are used to fit successive autoregressions of

fixed order p , under the linear null hypothesis the standardized one-step-ahead forecast

errors should be roughly identically and independently distributed with zero mean and unit

variance. But, from (1.18), once s begins to exceed m , the nonlinearity of the process should

cause systematic deviations in the forecast errors. Let the sum of the standardized one-step-ahead forecast errors be denoted by Z_s , where

$$Z_s = \sum_{i=s_{\min}}^s z_i \quad (s = 1, 2, \dots, n-p)$$

and z_i is the one-step-ahead forecast error. The cumulative sums can be plotted sequentially to give a graphical method for detecting nonlinearity and the location of the thresholds for the threshold models. To develop a test for linearity, Petrucci and Davis use an invariance principle for random walks. Let

$$T_n = \text{Max}_{s_{\min}+1 \leq s \leq n-p} |Z_s|$$

Then as $n \rightarrow \infty$,

$$\Pr(T_n / (n-p-s_{\min})^{1/2} \leq t) \rightarrow 4\pi^{-1} \sum_{k=0}^{\infty} (-1)^k (2k-1)^{-1} \exp\{-(2k+1)^2 \pi^2 / 8t^2\} \quad (1.19)$$

Consider testing the hypothesis $H_0: \beta_i^{(1)} = \beta_i^{(2)}$ ($i = 0, 1, 2, \dots, p$) against the alternative hypothesis that H_0 does not hold. Under the null, model (1.17) is linear and (1.19) holds for moderately large sample sizes. Let $1-p^*$ denote the value computed from the right side of equation (1.19) with t given by

$$t^* = \text{Max}_{s_{\min}+1 \leq s \leq n-p} |Z_s| / (n-p-s_{\min})^{1/2} \quad (1.20)$$

That is, p^* is the observed significance level of the test. The test rejects H_0 at significance level α if $p^* < \alpha$.

Tsay (1989) also uses arranged regressions to for test for threshold nonlinearity. His test is related to the nonlinearity test of Petrucci and Davis's (1986) in that it also makes use of the arranged regressions. Tsay noticed that in model (1.18), under the null of linearity,

the one-step-ahead standardized prediction residuals are white noise asymptotically and are orthogonal to the regressors $\{x_{(i)-d-l} \mid l = 1, 2, \dots, p\}$ for $i \leq m$. However, once $i > m$, the predictive residuals lose these properties. Consider the arranged regression based on (1.18). For fixed p and d , the effective number of observations in arranged regression is $(n-d-h+1)$, where n is the sample size, d is delay parameter, and $h = \max(1, p-d+1)$. Assume the recursive autoregressions begin with b observations so that there are $(n-d-b-h+1)$ predictive residuals available. Consider the least squares regression

$$\hat{e}_{(i)+d} = \phi_0 + \sum_{j=1}^p \phi_j x_{(i)+d-j} + \eta_{(i)+d} \quad (1.21)$$

where $\hat{e}_{(i)+d}$ is the standardized predicted residuals and $i = (b+1), \dots, (n-d-h+1)$. Since under the linear null $\hat{e}_{(i)+d}$ and $x_{(i)+d-j}$ ($j = 1, 2, \dots, p$) are orthogonal, the associated F statistic

$$\hat{F}(p, d) = \frac{\left(\sum \hat{e}_{(i)+d}^2 - \sum \hat{\eta}_{(i)+d}^2 \right) / (p+1)}{\sum \hat{\eta}_{(i)+d}^2 / (n-d-b-p-h)}$$

where the summations are over all of the observations in (1.21), should be distributed as $F_{(p+1, n-d-b-p-h)}$ under the null hypothesis that the parameters are the same across regimes.

Therefore, $\hat{F}_{(p,d)}$ can be used to test for possible threshold nonlinearity.

Tsay (1998) generalizes his univariate test for threshold nonlinearity to allow for multivariate series including cointegrated processes. If we want to test for nonlinearity in a $p \times 1$ vector y_t , consider the vector error correction model

$$\Delta y'_t = X'_{(t-1)} \Theta + \varepsilon'_t, \quad t = h+1, \dots, T$$

where $X'_{(t-1)} = (1, z_{t-d}, \Delta y_{t-1}, \dots, \Delta y_{t-k+1})$, $h = \max\{k, d\}$, $z_{t-d} = \beta' y_{t-d}$ is the threshold variable with known β , and Θ is a $(k+1) \times p$ matrix of coefficients. Consider the order statistic for S and denote the i^{th} smallest element of S by $z_{(i)}$. The arranged multivariate regression, ordered by the threshold variable z_{t-d} , will be

$$\Delta y'_{(i)+d} = X'_{(i)+d-1} \Theta + \varepsilon'_{(i)+d}, \quad i = 1, \dots, T-h.$$

Let $\hat{\Theta}_m$ denote the multivariate least squares estimate of Θ in the previous equation using the data from $i=1, 2, \dots, m$. Define

$$\hat{\varepsilon}_{(m+1)+d} = \Delta y_{(m+1)+d} - X'_{(m+1)+d-1} \hat{\Theta}_m$$

and

$$\hat{\xi}_{(m+1)+d} = \frac{\hat{\varepsilon}_{(m+1)+d}}{\left[1 + X'_{(m+1)+d-1} V_m X_{(m+1)+d}\right]^{\frac{1}{2}}}$$

where $V_m = \left(\sum_{i=1}^m X_{(i+1)+d-1} X'_{(i+1)+d-1}\right)$. Next, consider the multivariate regression

$$\hat{\xi}_{(m+1)+d} = X_{(m+1)+d-1} \Psi + \omega_{(m+1)+d}, \quad m = m_0 + 1, \dots, T-h \quad (1.22)$$

where m_0 denotes the starting point of the recursive least squares estimation. If there is no threshold nonlinearity, Ψ should be zero in (1.22) because the standardized residual $\hat{\xi}_{(m+1)+d}$ should be uncorrelated with the regressor $X_{(m+1)+d-1}$. To test the hypothesis $H_0: \Psi=0$ vs. $H_1:$

$\Psi \neq 0$, Tsay suggests using the test statistic

$$C(d) = \left(T - h - m_0 - (2(k-1) + 1)\right) \left\{ \ln(\det(S_0)) - \ln(\det(S_1)) \right\}$$

where

$$S_0 = \frac{1}{T-h-m_0} \hat{\xi}_{(l)+d} \hat{\xi}'_{(l)+d}$$

$$S_1 = \frac{1}{T-h-m_0} \hat{\omega}_{(l)+d} \hat{\omega}'_{(l)+d}$$

where $\hat{\omega}_{(l)+d}$ is the OLS residual from (1.22). Tsay claims that $C(d) \sim \chi^2_{2(k-1)}$ asymptotically.

To increase the power in detecting threshold effects, specification tests should be applied. However, a difficult issue associated with testing for a threshold effect is that conventional tests of the null of linear autoregressive model against the TAR alternative yield test statistics that have nonstandard distributions. Hansen (1999) considers a class of nested self-excited TAR (SETAR) models. Let Y_t be a univariate time series and let

$X_{(t-1)} = (1 \ Y_{t-1} \ \dots \ Y_{t-p})'$. A SETAR(g) model takes the form

$$Y_t = \alpha_1' X_{(t-1)} I_{1t}(\gamma, d) + \dots + \alpha_g' X_{(t-1)} I_{gt}(\gamma, d) + \varepsilon_t, \quad (1.23)$$

where $\gamma = (\gamma^{(1)}, \dots, \gamma^{(g-1)})$ and $I_{jt}(\gamma, d) = I_{jt}(\gamma^{(j-1)} < Y_{t-d} \leq \gamma^{(j)})$ with $j=1, 2, \dots, g$ and $-\infty =$

$\gamma^{(0)} < \gamma^{(1)} < \gamma^{(2)} < \dots < \gamma^{(g-1)} < \gamma^{(g)} = \infty$. Assume $d \leq p$, as usual. Suppose $\varepsilon_t \sim \text{i.i.d } N(0, \sigma^2)$. The

parameters in (1.23) can be collected as $\theta = (\alpha_1, \alpha_2, \dots, \alpha_g, \gamma, d)$. The least square estimator $\hat{\theta}$ solves the minimization problem

$$\hat{\theta} = \arg \min_{\theta} \sum_{t=1}^n \left(Y_t - \alpha_1' X_{t-1} I_{1t}(\gamma, d) - \dots - \alpha_g' X_{t-1} I_{gt}(\gamma, d) \right)^2 \quad (1.24)$$

Denote $\alpha = (\alpha_1' \ \alpha_2' \ \dots \ \alpha_g')$ and

$$X_{t-1}(\gamma, d) = \begin{pmatrix} X_{t-1}(\gamma, d)I_{1t}(\gamma, d) \\ \vdots \\ X_{t-1}(\gamma, d)I_{gt}(\gamma, d) \end{pmatrix}$$

Let $X(\gamma, d)$ be the $n \times g(p+1)$ matrix whose i^{th} row is $X_{t-1}(\gamma, d)'$ (p is the number of autoregressive lags in (1.23)). First, suppose that (γ, d) are known. The OLS estimator of α is

$$\hat{\alpha}(\gamma, d) = [X'(\gamma, d)X(\gamma, d)]^{-1} [X'(\gamma, d)Y] . \quad (1.25)$$

Let

$$S_g(\gamma, d) = (Y - X(\gamma, d)\hat{\alpha})'(Y - X(\gamma, d)\hat{\alpha}) \quad (1.26)$$

be the residual sum of squared errors for given (γ, d) . For notational simplicity, let S_g stand for $S_g(\gamma, d)$. To test the hypothesis that the model in (1.23) has k regimes instead of g ($1 \leq k < g$), Hansen suggests using

$$F_{kg} = n \left[\frac{S_k - S_g}{S_g} \right]$$

which has a standard chi-square asymptotic distribution with degree of freedom $(g-k)(p+1)$.

If F_{kg} is large enough, reject the null that the model has k regimes. This is the likelihood ratio test because the errors are normally distributed.

However, (γ, d) are generally unknown. Hansen suggests getting the estimate

$$(\hat{\gamma}, \hat{d}) = \arg \min_{\gamma, d} S_g(\gamma, d) \quad (1.27)$$

Once the solution to (1.27) is found (by grid search), substitute it into equation (1.25) to get $\hat{\alpha}(\hat{\gamma}, \hat{d})$. Substitute $\hat{\alpha}(\hat{\gamma}, \hat{d})$ into (1.26) to get $S_g(\hat{\gamma}, \hat{d})$. Then the F test becomes

$$F_{kg} = n \left[\frac{S_k(\hat{\gamma}, \hat{d}) - S_g(\hat{\gamma}, \hat{d})}{S_g(\hat{\gamma}, \hat{d})} \right] \quad (1.28)$$

The F_{kg} in (1.28) has a non-standard asymptotic distribution and Hansen suggests using a bootstrap method to get critical values.

1.3.2 Literature on Threshold Cointegration

Balke and Fomby (1997) propose the concept of “threshold cointegration”, which is a link between cointegration and threshold models in time series. In threshold cointegration, cointegration is the global characteristic of the time series and the threshold regimes are local behavior. They first consider a simple bivariate system (y_t, x_t) such that

$$y_t - \alpha x_t = z_t, \quad \text{where } z_t = \rho^{(i)} z_{t-1} + \varepsilon_t \quad (1.29)$$

$$y_t - \beta x_t = B_t, \quad \text{where } B_t = B_{t-1} + \eta_t \quad (1.30)$$

ε_t and η_t are i.i.d. zero-mean variables. In this system, (1.29) defines a long-term equilibrium because $y_t - \alpha x_t$ is mean reverting. Any other linear combination, including $y_t - \beta x_t$ in (1.30), are nonstationary. Balke and Fomby define

$$\rho^{(i)} = \begin{cases} 1 & \text{if } |z_{t-1}| \leq \gamma \\ \rho, \text{ with } |\rho| < 1 & \text{if } |z_{t-1}| > \gamma \end{cases}$$

where γ is a critical threshold. In this system, as long as $|z_{t-1}| \leq \gamma$, z_t follows random walk and there is not a tendency for the system to go back to the equilibrium relationship. But once the threshold is reached, that is, $|z_{t-1}| > \gamma$, the system will exhibit a tendency to drift back to equilibrium. This is the “Equilibrium-TAR”, which can be better illustrated in Figure 3. The

process tends to return to the equilibrium $y_t - \alpha x_t = 0$ when outside the band. But once within, the system follows random walk.

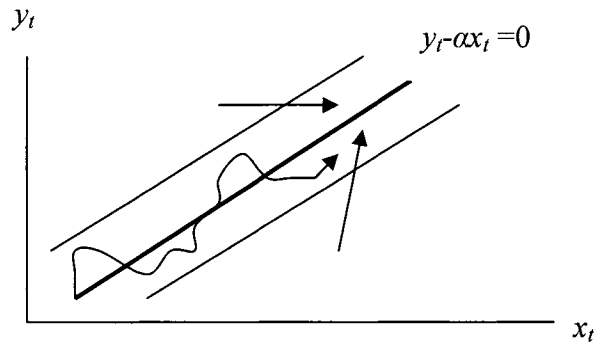


Figure 3: The Equilibrium-TAR Model

Another example of threshold cointegration is the “Band-TAR”, which is similar to the Equilibrium-TAR except that the path of adjustment to the equilibrium is different. In a Band-TAR, the equilibrium error z_t is defined as

$$z_t = \begin{cases} \theta(1 - \rho) + \rho z_{t-1} + \varepsilon_t & \text{if } z_{t-1} > \gamma \\ z_{t-1} & \text{if } -\gamma \leq z_{t-1} \leq \gamma \\ -\theta(1 - \rho) + \rho z_{t-1} + \varepsilon_t & \text{if } z_{t-1} < -\gamma \end{cases}$$

The BAND-TAR is illustrated in Figure 4. If the process is outside the band, it will return to the boundaries of the band instead of the equilibrium $y_t - \alpha x_t = 0$. But once inside the band, the process follows a random walk.

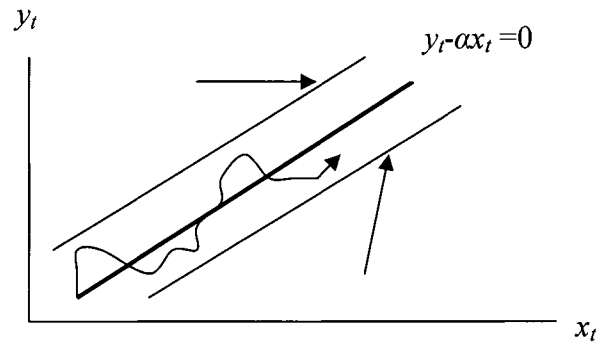


Figure 4: The Band-TAR Model

To test for threshold cointegration, Balke and Fomby suggest a two step procedure. First, test for cointegration. The standard tests for cointegration in linear time models turn out to work asymptotically in the threshold cointegration setting. Therefore, either Engle and Granger's two-stage procedure or Johansen's full information maximum likelihood approach can be applied. Next, based on the estimated cointegrating vector, the residual sequence $z_t = y_t - \hat{\alpha}x_t$ is used to test for nonlinearity. The tests for a threshold effect discussed before, including the methods described in Petrucci and Davis (1986), Tsay (1989) and Hansen (1999), can be applied. Based on their Monte Carlo experiments, Balke and Fomby find that the performance of this two-step procedure is satisfactory.

Hansen and Seo (2002) consider a two-regime threshold cointegration model

$$\Delta x_t = \begin{cases} A_1' X_{t-1}(\beta) + \varepsilon_t & \text{if } z_{t-1} \leq \gamma \\ A_2' X_{t-1}(\beta) + \varepsilon_t & \text{if } z_{t-1} > \gamma \end{cases}$$

where x_t is a vector of random variables and

$$X_{t-1}(\beta) = \begin{pmatrix} 1 \\ z_{t-1}(\beta) \\ \Delta x_{t-1} \\ \Delta x_{t-1} \\ \vdots \\ \Delta x_{t-k} \end{pmatrix}$$

This can be written compactly as

$$\Delta x_t = A'_1 X_{t-1}(\beta) d_{1t}(\beta, \gamma) + A'_2 X_{t-1}(\beta) d_{2t}(\beta, \gamma) + \varepsilon_t \quad (1.31)$$

where

$$d_{1t}(\beta, \gamma) = I(z_{t-1}(\beta) \leq \gamma)$$

$$d_{2t}(\beta, \gamma) = I(z_{t-1}(\beta) > \gamma)$$

$I(\bullet)$ is the indicator function, $\varepsilon_t \sim$ i.i.d. $N(0, \Sigma)$. For simplicity, define

$$z_{t-1}(\beta) = x'_{t-1} \beta.$$

The threshold effect only has content if $0 < \Pr(z_{t-1}(\beta) \leq \gamma) < 1$, because otherwise the model simplifies to linear cointegration. Hansen and Seo propose a maximum likelihood algorithm for estimating this model and a *SupLM* statistic to test the null hypothesis of one regime versus the alternative of two regimes. Details on their procedure are presented in Section 4.1.

Lo and Zivot (2000) study a three-regime bivariate vector threshold error correction model. It is a natural generalization of the two-regime TVECM of Hansen and Seo (2002). In this dissertation, however, we are going to focus on the two-regime TVECM only.

1.3.3 The Fisher Effect and Threshold Cointegration

As stated before, it would be appropriate to apply the threshold cointegration model proposed by Balke and Fomby (1997) to test for the Fisher effect. This paper is filling several blanks in the empirical literature.

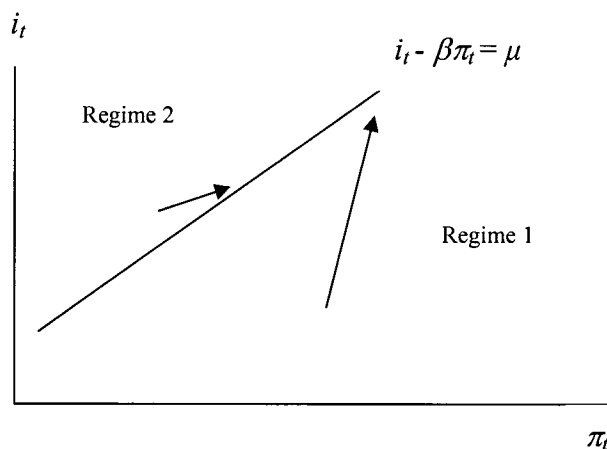
1. *Testing the stationarity of the nominal interest rate and the inflation rate under the TAR alternative.* If the nominal interest rate and the inflation rate follow a TAR process, the standard unit root tests, e.g., the ADF test and Phillips and Perron test, are misspecified. The result of this misspecification is that the standard unit root tests will have low power. Therefore, if the series being tested follows a TAR process, the standard unit root tests may fail to reject the null hypothesis that the sequence contains a unit root. Testing the stationarity of the nominal interest rates and inflation rate under the TAR alternative, however, will not suffer from this kind of problem. Therefore, we are going to apply the Enders and Granger (1998) test to check the order of integration for the nominal interest rate and the inflation.
2. *Testing threshold cointegration between the nominal interest rate and the inflation rate.* According to our study, both the nominal interest rate and the inflation rate are $I(1)$, and the two variables are cointegrated. Previous studies have found similar results, although the estimates of the cointegrating vector vary

across different studies and therefore the conclusions differ. However, nearly all previous studies are done under the linear cointegration framework. It is possible that the adjustment path is asymmetric and the two variables are threshold cointegrated, as in Balke and Fomby (1997). To be more specific, the model may have more than one regime, determined by the previous period's equilibrium error. The equilibrium relationship is constant across different regimes but the adjustment speeds are different. In this dissertation, threshold cointegration tests will be carried out to detect this kind of nonlinear behavior, following the procedure described by Balke and Fomby (1997).

3. *Modeling the Fisher relationship in a TVECM framework.* If nonlinearity is detected, a two-regime threshold vector error correction similar model to Hansen and Seo (2002) will be applied to capture the nonlinear feature of the relationship between the nominal interest rate and the inflation rate. The cointegrating vector, which is the long-run equilibrium in the system, will be constant but the adjustment speed differs across regimes. The threshold variable is the equilibrium error in the previous period. If the Fisher equation holds, we would expect the cointegrating vector to be (1,-1) in both regimes, ignoring the tax effect .

The setup is better illustrated in Figure 2, which is presented again below. The solid line in it, $i_t - \beta\pi_t = \mu$, is the long-run equilibrium defined by the cointegrating vector between

inflation and the nominal interest rate. Here we assume the threshold is the real interest rate. In Regime 1, the inflation rate is relatively high but the nominal interest rate is relatively low, which implies that the real interest rate is below the long-run equilibrium. If so, the adjustment towards the equilibrium, $i_t - \beta\pi_t = \mu$, is pretty fast. However, if the economy is in Regime 2, in which the real interest rate is above the equilibrium, the adjustment speed tends to be slower than in Regime 1. Hansen and Seo (2000) estimation procedure can be applied to fit this model and to test for threshold behavior.



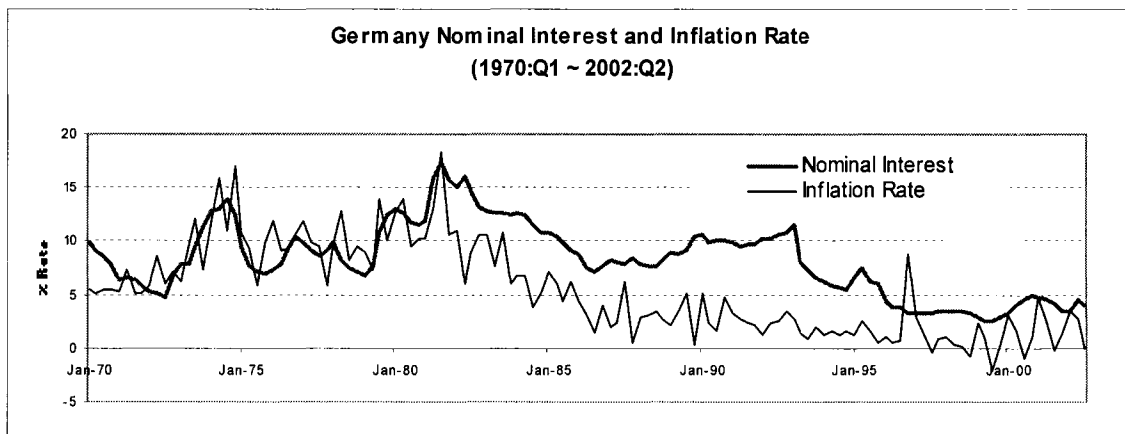
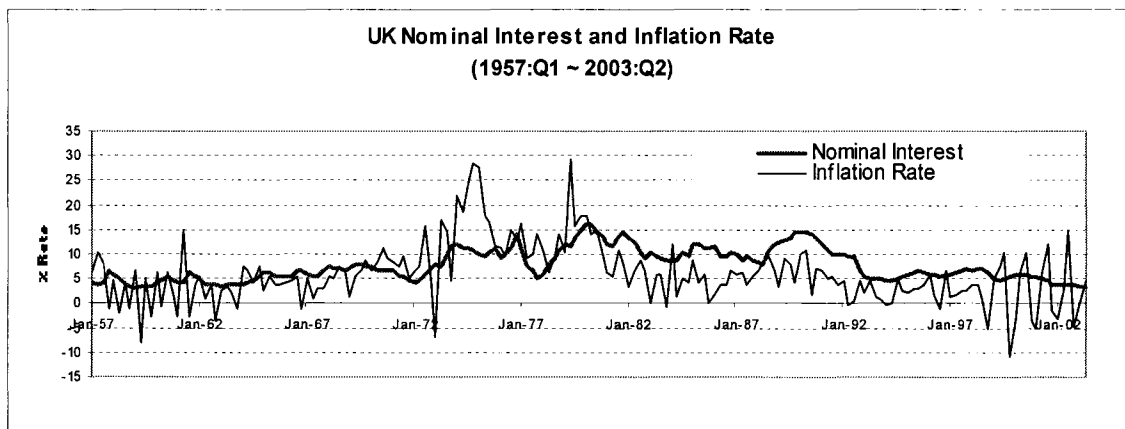
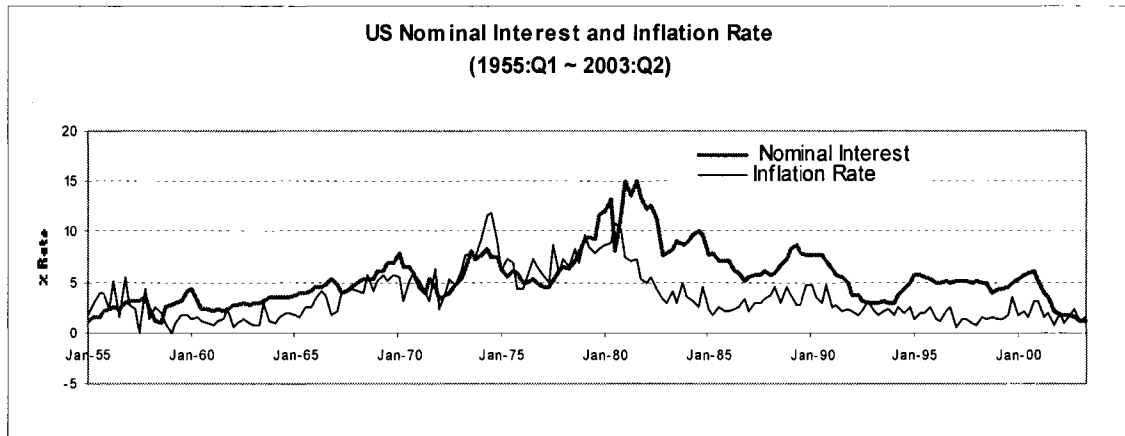
Accordingly, the next three chapters in this dissertation will be devoted to the three topics above. In Chapter Two, we will check the order of integration of the nominal interest rate and the inflation rate with linear and nonlinear models. In Chapter Three, we will test for threshold cointegration between the nominal interest rate and the inflation rate. In Chapter Four, a two-regime threshold error correction model with asymmetric adjustment path to the long-run equilibrium will be estimated, following the procedure in Hansen and Seo (2002). Chapter Five provides conclusions and directions for future research.

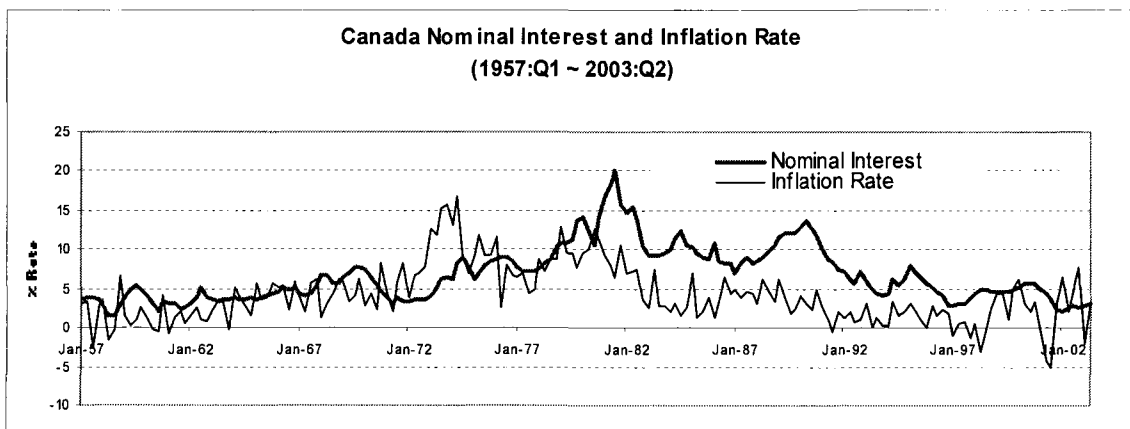
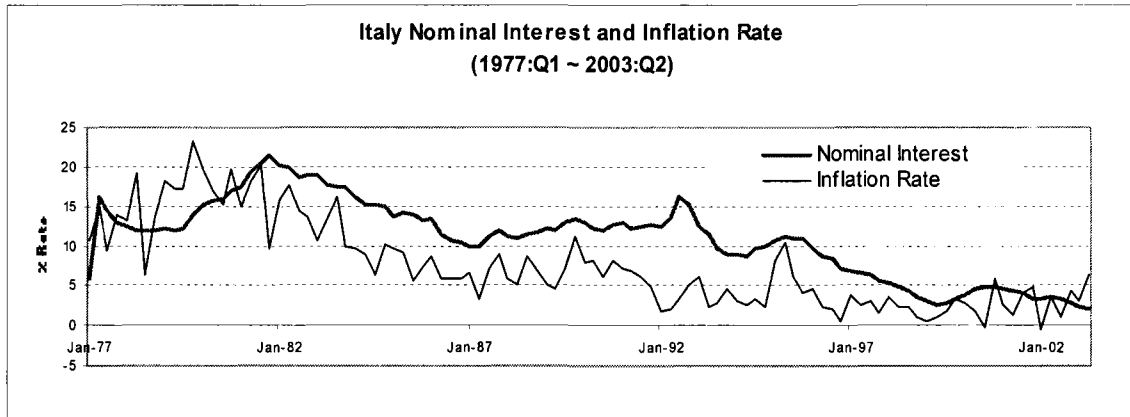
1.4 Description of Data

Throughout our analysis, quarterly data will be used. The 3-month Treasury Bill rate and the differenced log of seasonally adjusted GDP Implicit Price Deflator will be used as the nominal interest rate and inflation rate, respectively. Both the nominal interest rate and the inflation rate are annualized. The countries being studied include the United States, the United Kingdom, Germany, Italy and Canada.

For the United States, the data set is from the database of Federal Reserve Bank of St. Louis and the sample period is 1955:Q1 ~ 2003:Q2. For the other countries, the data set is from the International Financial Statistics (IFS) data base published by the International Monetary Fund (IMF), and the periods of study 1957:Q1~2003:Q2, 1972:Q1~2003:Q2, 1977Q1~2003:Q2, 1957:Q1~2003:Q2 for the United Kingdom, Germany, Italy and Canada, respectively. The reason for including these countries in the study is that their 3-month Treasury bill rate and the seasonally adjusted GDP Deflator are available and the data go as far back as 1970s. The nominal interest rate and the inflation rate for the countries under study are plotted below.

Figure 5: Plot of Nominal Interest Rates and Inflation Rates
Of Selected Countries





CHAPTER TWO: UNIT ROOT TESTS

2.1 Linear Unit Root Tests

Many economic variables contain a unit root. To test for a unit root, consider the standard regression

$$y_t = \beta y_{t-1} + \varepsilon_t \quad (2.1)$$

where y_{t-1} is uncorrelated with ε_t . Generally, to test the hypothesis that $\beta = \beta_0$ with $-1 < \beta_0 < 1$, we can use the standard t -test. However, when β_0 approaches one, the usual t -test will become invalid¹. To test the hypothesis that $\beta = 1$, Dickey and Fuller (1979) propose the well-known Dickey-Fuller test. In the test, first run the regression

$$\Delta y_t = \gamma y_{t-1} + \varepsilon_t \quad (2.2)$$

where the disturbances are independent and have a constant variance. Comparing Equation (2.1) and (2.2), we can see that testing $\beta = 1$ in (2.1) is equivalent to testing $\gamma = 0$ in (2.2), which can be done by comparing the t -value of γ with the critical values provided by Dickey and Fuller. To address possible autocorrelation in the sequence ε_t , more autoregressive lags in Δy_t can be added to (2.2):

$$\Delta y_t = \gamma y_{t-1} + \sum_{i=1}^p \beta_i \Delta y_{t-i} + \varepsilon_t \quad (2.3)$$

Adding more autoregressive lags does not change the critical values of the t -statistic of γ .

This procedure is called the Augmented Dickey-Fuller test (ADF). An intercept and/or a

¹ When β approaches ± 1 , its t -statistic will follow a nonstandard distribution asymptotically, according to Dickey and Fuller (1979).

time trend can be added to (2.2) and (2.3), but the t -value of γ should be compared with different critical values in each of these cases.

The Dickey-Fuller test and ADF test have been applied in many fields. However, the common assumption in the two tests that ε_t are independent and identically distributed may be too strict in practice, and there are no explicit rules on how many autoregressive lags (p in Equation (2.3)) should be included. At the same time, as pointed out in Ng and Perron (2001), the tests have low power when the root of the autoregressive polynomial is close to but less than unity.

To address these issues, econometricians have developed numerous alternative procedures to test the presence of unit roots in a univariate time series. Phillips and Perron (1988) develop a generalization of the Dickey-Fuller test which allows for milder assumptions concerning the distribution of ε_t . Elliott, Rothenberg and Stock (1996) propose a modified version of ADF test (hereafter ADF^{GLS}) to improve the power of ADF test with an intercept and/or a time trend in the data. Elliott, Rothenberg and Stock (1996) find that the ADF^{GLS} has substantially improved the power when unknown deterministic components are present, and that the modified test works well in small samples. Ng and Perron(1996, 2001) state the fact that when there are errors with a moving-average root close to -1, a high order augmented autoregression is necessary for unit root tests to have a good size, but the AIC and BIC lag selection criteria tend to select a truncation lag that is too small. They suggest using a Modified Information Criteria (MIC) when selecting the autoregressive lag

length, and claim that the ADF^{GLS} test with lag length determined by MIC has good size and power.

We first test the order of integration by applying the ADF test to nominal interest rate and the inflation rate. The results are presented in Table 1.

Table 1: ADF test on the Nominal Interest Rate and the Inflation Rate

	Country	ADF (no intercept or trend)		ADF(with intercept, no trend)	
		Interest	Inflation	Interest	Inflation
		τ	τ	τ_{μ}	τ_{μ}
Level	US	-0.91	-1.07	-2.04	-1.60
	UK	-0.98	-1.65*	-2.65*	-2.54
	Germany	-1.08	-1.55	-1.97	-2.86*
	Italy	-1.03	-1.16	-0.51	-1.45
	Canada	-0.96	-1.47	-2.43	-2.62
First Difference	US	-10.54***	-12.19***	-10.51***	-12.16***
	UK	-6.88***	-6.39***	-6.86***	-6.37***
	Germany	-5.44***	-10.96***	-5.44***	-10.92***
	Italy	-6.42***	-12.15***	-6.48***	-12.10***
	Canada	-9.75***	-17.48***	-9.72***	-17.43***

*: significant at 10% **: significant at 5% ***: significant at 1%

Note:

- 1) For the category “Without Intercept or Time Trend”, the regression equation is

$$\Delta y_t = \gamma y_{t-1} + \sum_{i=1}^p \beta_i \Delta y_{t-i} + \varepsilon_t \quad (\text{a});$$

for “With Intercept, No Time Trend”, the regression is $\Delta y_t = \mu + \gamma y_{t-1} + \sum_{i=1}^p \beta_i \Delta y_{t-i} + \varepsilon_t \quad (\text{b}).$

- 2) The statistics labeled as τ and τ_{μ} are the corresponding statistics to use for equations (a) and (b), respectively.

- 3) The number of lags (p in equation (a) and (b)) for the ADF test is selected by minimizing AIC.
- 4) For τ statistic, the critical values are -2.58 (1%), -1.95(5%) and -1.62(10%); for τ_{μ} statistic, -3.46 (1%), -2.88(5%) and -2.57(10%).

According to the ADF test, none of the test statistics for the levels are significant at the 5% or 1% significance level. At the 10% significance level, only the τ statistic for the UK inflation rate, τ_{μ} for the UK nominal interest rate and τ_{μ} for the Germany inflation rate are significant. Therefore it seems there is strong evidence that both the nominal interest and inflation rate contain at least one unit root. A second unit root is strongly rejected because all of the test statistics for the first difference are significant at 1% level. Accordingly, based on ADF test, both the nominal interest and the inflation rate appear to be $I(1)$.

As pointed out by Elliott, Rothenberg and Stock (1996), the ADF test may have low power when unknown deterministic components are present. Hence ADF test may fail to reject the null hypothesis sufficiently often if the series to be tested comprises an unknown deterministic component (an intercept, for example) and a stationary disturbance process. In our case, the ADF test may fail to reject the null hypothesis because of possible deterministic components in the nominal interest rate and the inflation rate. To make sure that this is not the case, we have also applied the ADF^{GLS} test to the nominal interest rate and inflation, detrended first with an intercept described in Elliott, Rothenberg and Stock (1996). The optimal lag length is determined by minimizing the Modified Information Criteria (MIC) as in Ng and Perron(2001). The results can be found in Table 2.

Table 2: ADF^{GLS} Unit Root Test on Nominal Interest rate
And Inflation Rate

Country	Nominal Interest Rate	Inflation Rate
US	-1.35	-1.40
UK	-1.91*	-1.87*
Germany	-1.98**	-0.72
Italy	-0.94	-1.15
Canada	-1.60	-2.37**

*: significant at 10% **: significant at 5% ***: significant at 1%

Note:

- 1) The test statistic is from the equation $\Delta y_t = \gamma y_{t-1} + \sum_{i=1}^p \beta_i \Delta y_{t-i} + \varepsilon_t$, where y_t is the GLS detrended series (with an intercept) and p is selected minimizing Modified Information Criteria as in Ng and Perron (2001).
- 2) The critical values are -2.58 (1%), -1.95(5%) and -1.62(10%).

According to the results of ADF^{GLS} test, none of the test statistics are significant at the 1% significance level; at the 5% level the null hypothesis was rejected for the Canada inflation rate only; at the 10% level the null hypothesis was rejected for the UK nominal interest rate, the UK inflation rate and Germany nominal interest rate and the Canada inflation rate. Therefore, there is only mild evidence against the null hypothesis that both nominal interest rate and inflation rate are integrated of order one.

2.2 Unit Root Test under the Threshold Alternative

From (2.2) and (2.3), we can see that if y_t is not linear, the ADF test is misspecified under the alternative. The consequence of the misspecification is that the power of the test is distorted, and it fails to reject the null hypothesis sufficiently often. This problem has been addressed by Enders and Granger (1998) and Enders (2001).

Enders and Granger (1998) study unit root tests under threshold alternatives. Consider the alternative specification to (2.1),

$$\Delta y_t = I_t \rho_1 (y_{t-1} - \tau) + (1 - I_t) \rho_2 (y_{t-1} - \tau) + \varepsilon_t \quad (2.4)$$

The indicator function I_t can be specified in two ways. First, it can be defined as

$$I_t = \begin{cases} 1 & \text{if } y_{t-1} \geq \tau \\ 0 & \text{if } y_{t-1} < \tau \end{cases} \quad (2.5)$$

where τ is a threshold to be estimated. The model defined in Equation (2.4) and (2.5) is called a threshold autoregressive (TAR) model, in which the speed of mean-reverting behavior depends on y_{t-1} , the level of the sequence in the prior period. Another possible way of defining I_t is

$$I_t = \begin{cases} 1 & \text{if } \Delta y_{t-1} \geq \tau \\ 0 & \text{if } \Delta y_{t-1} < \tau \end{cases} \quad (2.6)$$

The model as defined in Equation (2.4) and (2.6) is called the momentum threshold autoregressive (M-TAR) model, in which the speed of mean-reverting behavior depends on Δy_{t-1} , the first difference in the prior period. To test for a possible unit root, Enders and Granger (1998) propose the following procedure:

- Demean y_t first and save the residuals as \hat{y}_t . Set the indicator function I_t in (2.5) (or (2.6)) according to whether \hat{y}_t (or $\Delta\hat{y}_t$) is positive or negative. Run a regression in the form of (2.4) and look at the F -statistic for the null hypothesis $\rho_1=\rho_2=0$. If the F -statistic is larger than the critical values tabulated in Enders and Granger (1998), the null hypothesis of unit root should be rejected. The F -statistic from the TAR (or M-TAR) model is called the Φ_μ (or Φ_μ^M) statistic.
- Check the estimated residual series ε_t from (2.4) to verify if it can be reasonably characterized by a white-noise process. If not, re-estimate the model in the form

$$\Delta\hat{y}_t = I_t\rho_1\hat{y}_{t-1} + (1-I_t)\rho_2\hat{y}_{t-1} + \sum_{i=1}^p \beta_i\Delta\hat{y}_{t-i} + \varepsilon_t. \quad (2.7)$$

Lag lengths can be determined using AIC or BIC.

By demeaning y_t , the Enders-Granger procedure is actually estimating equation (2.4) in the form

$$\Delta y_t = I_t\rho_1(y_{t-1} - \bar{y}) - (1-I_t)\rho_2(y_{t-1} - \bar{y}) + \varepsilon_t$$

where \bar{y} is the sample mean. However, if the adjustment is asymmetric, the sample mean is a biased estimate of τ , as pointed out by Enders (2001). To obtain a consistent estimate of the threshold, Enders (2001) suggests using the procedure described in Chan (1993), which looks at the ordered values of the y_t sequence, denoted by $y^{(1)} \leq y^{(2)} \leq \dots \leq y^{(i)} \leq \dots \leq y^{(T)}$. For each value of $y^{(i)}$, set $\tau = y^{(i)}$ and estimate an equation in the form of (2.4). The regression with the smallest residual sum of squares contains the consistent estimate of the threshold.

The corresponding Φ_μ (or Φ_μ^M) statistic can be obtained and compared with the tabulated critical values in Enders (2001). In practice, the highest and lowest 15% of the $y^{(i)}$ series are excluded from the grid search to ensure an adequate number of observations in each regime.

With the 3-month T-Bill rates and inflation rates from the United States, the United Kingdom, Germany, Italy and Canada, we have carried out the Enders-Granger TAR and M-TAR tests, in which the threshold estimates are obtained by following Enders (2001). The results are presented in Table 3.

Table 3: Enders and Granger TAR and M-TAR Unit Root Test

	Nominal Interest Rate		The Inflation Rate	
	Φ_μ	Φ_μ^M	Φ_μ	Φ_μ^M
US	5.12	2.86	2.67	5.96 **
UK	3.95	1.57	5.09	5.01 *
Germany	6.10 **	2.59	2.43	3.45
Italy	1.27	1.57	2.47	2.04
Canada	3.93	5.13 *	3.36	4.93 *

*: significant at 10% **: significant at 5% ***: significant at 1%

Note:

1. The regression equation used to test for the presence of a unit root is

$$\Delta y_t = I_1 \rho_1 (y_t - \tau) + (1 - I_1) \rho_2 (y_t - \tau) + \sum_{i=1}^p \beta_i \Delta y_{t-i} + \varepsilon_t$$

with I_t defined in equation (2.5) or (2.6).

2. The null hypothesis is $\rho_1 = \rho_2 = 0$. The test statistics labeled as Φ_{μ} and Φ_{μ}^M are the corresponding F -statistic from the TAR and M-TAR model, respectively.
3. The number of lags is selected by minimizing AIC.

By Enders and Granger TAR and M-TAR test, it seems a unit root can not be rejected for most of countries under our study, which is consistent with the conclusions from the ADF and Ng-Perron tests.

2.3 End-of-chapter Summary

For most of the countries under our study, the existence of a unit root in the nominal interest rate and the inflation rate is not rejected by the ADF test, but a second unit root is firmly rejected. To ensure that the conclusion is not erroneously drawn due to possibly low power of ADF test, the Ng-Perron test and Ender-Granger test have been applied to the series. Compared with ADF test, the Ng-Perron procedure may have higher power when there is an unknown deterministic component, and the Enders-Granger test will not suffer from power distortion when nonlinearity is present. The results of both tests seem to be consistent with the ADF.

CHAPTER THREE: COINTEGRATION TESTS

3.1 Linear Cointegration Tests

Both the nominal interest rate and inflation are integrated of order one, according to Chapter Two. If the Fisher relationship holds, there exists a $\beta \geq 1$ such that $i_t - \beta\pi_t$ is a stationary process, which can be modeled in time series by cointegration. There are two classes of cointegration analyses: linear and nonlinear models. The linear cointegration models are linear under both the null and the alternative, for example, Engle and Granger (1987), Johansen (1988) and Phillips-Ouliaris (1990). Nonlinear cointegration models, however, are linear under the null but nonlinear under the alternative, for example, Hansen and Seo (2002) and Lo and Zivot (2000).

Linear cointegration analyses including Johansen (1998) and Phillips-Ouliaris (1990) P_z test will be estimated in this section. A nonlinear cointegration model, Hansen and Seo (2000) TVECM, will be studied in Chapter Four.

The Johansen (1988) cointegration test is based on a vector autoregressive model, which can be represented by

$$\Delta y_t = A_0 + \Pi y_{t-1} + \sum_{i=1}^p A_i \Delta y_{t-i} + \varepsilon_t \quad (3.1)$$

where $y_t = (i_t, \pi_t)'$ and $\varepsilon_t \sim \text{i.i.d } N(0, \Sigma)$. The key feature of this test is to look at the rank of the coefficient matrix Π . If $\text{rank}(\Pi) = 0$, i_t and π_t are both $I(1)$ but not cointegrated. If $\text{rank}(\Pi) = 1$, they are cointegrated. If $\text{rank}(\Pi) = 2$, they are both stationary. However, the $\text{rank}(\Pi) = 2$ case can be ruled out from our study because both i_t and π_t are $I(1)$ according to the unit root tests in Chapter Two.

To test the null hypothesis of no cointegration against the existence of cointegration, we can look at the estimated eigenvalues of Π , $\hat{\lambda}_1$ and $\hat{\lambda}_2$ ($\hat{\lambda}_1 > \hat{\lambda}_2$).

Consider the following test statistic proposed by Johansen (1988)

$$\lambda_{\text{trace}}(0) = -T \sum_{i=1}^2 \ln(1 - \hat{\lambda}_i).$$

The $\lambda_{\text{trace}}(0)$ statistic can be used to test the null hypothesis $\text{rank}(\Pi) = 0$ against the alternative $\text{rank}(\Pi) \neq 0$. Another test statistic proposed by Johansen (1988) is

$$\lambda_{\text{max}}(0, 1) = -T \ln(1 - \hat{\lambda}_1).$$

The $\lambda_{\text{max}}(0, 1)$ statistic can be used to test the null hypothesis $\text{rank}(\Pi) = 0$ against the alternative $\text{rank}(\Pi) = 1$. As we have mentioned before, the $\text{rank}(\Pi) = 2$ case can be ruled out from our study. Therefore, we are going to use the $\lambda_{\text{max}}(0, 1)$ test only later on.

Phillips-Ouliaris (1990) P_z test looks at the residuals instead of the coefficient matrix Π . To be more exact, consider the vector multivariate least squares regression

$$y_t = \Pi y_{t-1} + \zeta_t \tag{3.2}$$

where $y_t = (i_t, \pi_t)'$. Let $\hat{\zeta}_t$ be the OLS residuals from (3.2). The heteroskedasticity and autocorrelation consistent (HAC) estimator of the covariance matrix of ζ_t is

$$\hat{\Omega} = \frac{1}{T} \sum_{t=1}^T \hat{\zeta}_t \hat{\zeta}_t' + \frac{1}{T} \sum_{s=1}^l w_{sl} \sum_{t=s+1}^T \left(\hat{\zeta}_t \hat{\zeta}_{t-s}' + \hat{\zeta}_{t-s} \hat{\zeta}_t' \right)$$

for some choice of lag window l (see Andrews (1990)) and weights w_{sl} (for example,

$w_{sl} = 1 - s/(l+1)$, see Newey and West (1987)). The multivariate P_z trace statistic is defined

as

$$P_z = T \operatorname{tr} \left(\hat{\Omega} M_{zz}^{-1} \right) \quad \text{with} \quad M_{zz} = \frac{1}{T} \sum_{t=1}^T y_t y_t'.$$

The P_z statistic is constructed as Hotelling's T -square statistic, which is a common statistic in multivariate analysis for tests of multivariate dispersion. The critical values for the P_z statistic are tabulated in Phillips-Ouliaris (1990).

To verify the existence of the Fisher effect, the Johansen (1988) and Phillips-Ouliaris (1990) procedures have been applied to the nominal interest rate and inflation of the United States, the United Kingdom, Germany, Italy and Canada (the detailed description of the dataset can be found in Section 1.4). The test results are presented in Table 4.

Table 4: Johansen (1988) and Phillips-Ouliaris (1990) Test of Cointegration

Country	Johansen Maximum Likelihood Ratio Cointegration Test		Phillips-Ouliaris Multivariate P_z Cointegration Test	
	β	$\lambda_{\max}(0,1)$	β	P_z
US	1.27	18.77 ***	1.46	65.09**
UK	0.90	41.05 ***	0.96	165.68***
Germany	1.59	38.27 ***	1.77	132.66***
Italy	1.24	10.74	1.33	46.73
Canada	1.38	17.62 **	1.47	84.53 ***

*: significant at 10% **: significant at 5% ***: significant at 1%

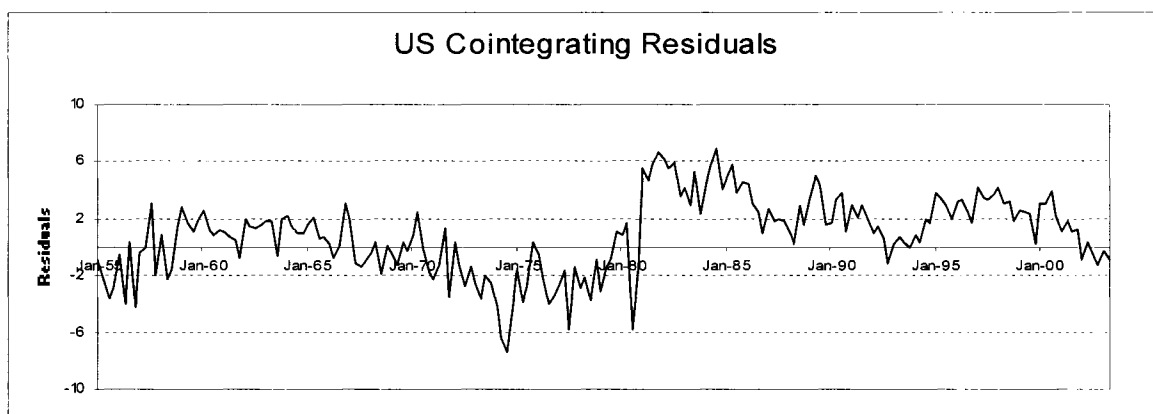
Note:

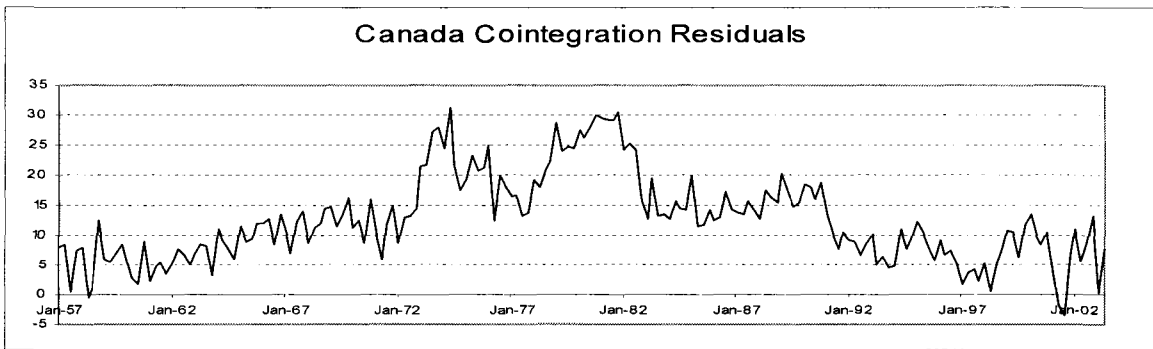
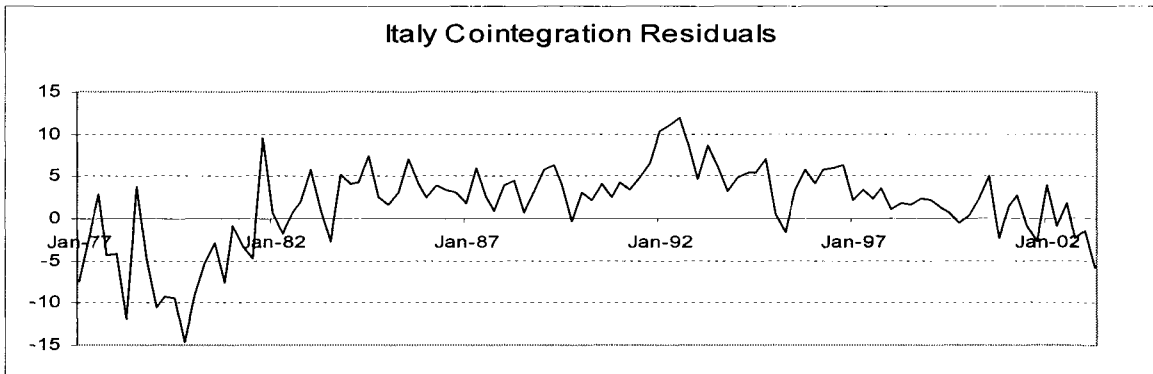
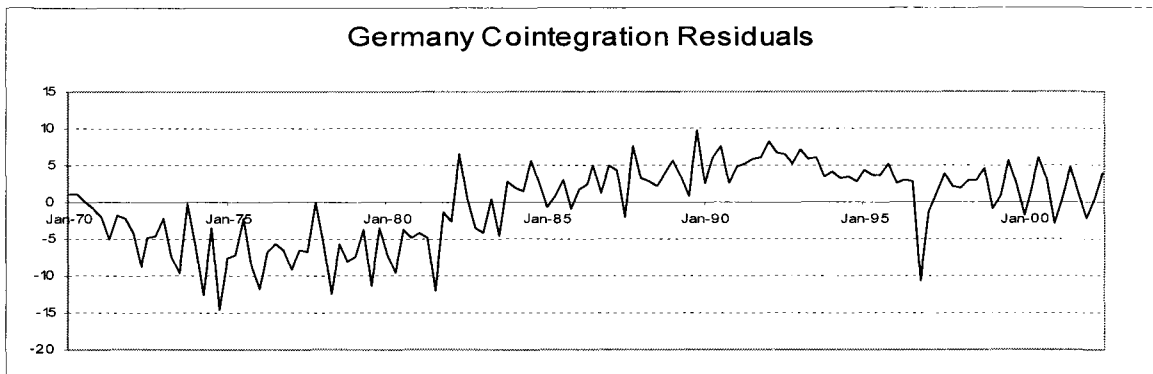
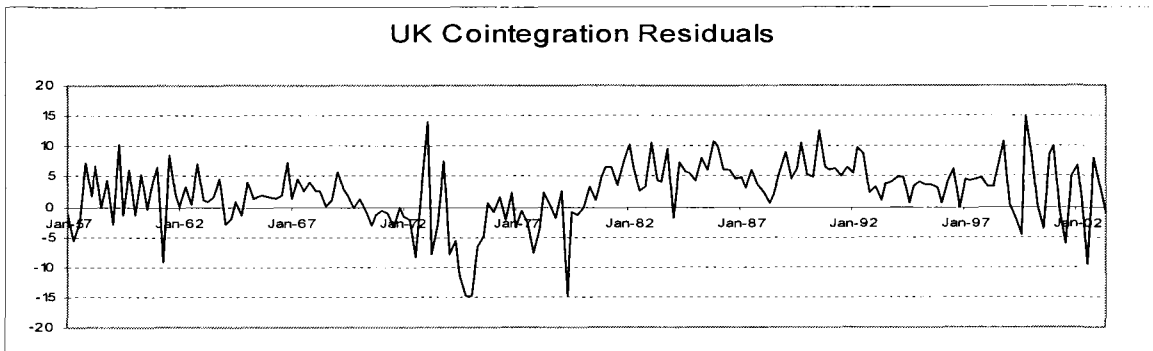
- 1) For both tests, there is an intercept in model ((3.1) and (3.2)).
- 2) For Johansen test, the number of autoregressive lags is selected by minimizing AIC; for Phillips-Ouliaris test, automatic window size is used as suggested in Andrews (1991).
- 3) The null hypothesis for both Johansen and Phillips-Ouliaris test is that there is no cointegration.

Both the Johansen $\lambda_{\max}(0,1)$ test and Phillips-Ouliaris multivariate P_z test reject the null hypothesis of no cointegration for US, UK, Germany and Canada at the 5% level. The only exception is Italy, for which we fail to reject the null hypothesis of no cointegration. This may be due to the relatively small size of the Italy dataset. In the IFS database, the 3-month T-Bill rate of Italy is not available until after the first quarter of 1977, which is the shortest sample period of all the countries under study.

For both Johansen and Phillips-Ouliaris procedure, the estimated Fisher effect β is greater than one for all the countries except the United Kingdom. With the estimated β , we can compute the cointegration residuals, which are the estimated deviations from the long-term attractor. The plot of estimated residuals from Johansen and Phillips-Ouliaris procedure are given in Figure 6 and Appendix C, respectively. One thing worth noting is that for the countries under study, the system tends to be below the long-term equilibrium in most of 1970s but above the equilibrium in most of 1980s. If we look back, almost all of the countries in our study experienced high inflation in most 1970s, especially the United States, due to hikes in oil prices. In the 1980s, however, governments changed their monetary policy and inflation was kept down. Therefore, we suspect that the sustained deviation from the long-term equilibrium can be partly explained by changes in government monetary policy.

Figure 6: Plot Cointegration Residuals from Johansen
Maximum Likelihood Procedure





According to the Johansen (1988) procedure, the estimated coefficient β is greater than one for all the countries except the United Kingdom. To verify the existence of a Fisher effect, we will apply the Johansen and Juselius (1990) likelihood ratio test of the hypothesis $\beta=1$. The likelihood ratio test statistics and their corresponding p-values are given in Table 5.

Table 5: The Likelihood Ratio Test of $\beta=1$ Based on the Johansen and Juselius (1990)

	Estimated β	LR Statistic	P-Value
US	1.27 (0.32)	0.95	0.33
UK	0.90 (0.19)	0.22	0.64
Germany	1.59 (0.27)	8.51	0.00
Canada	1.38 (0.26)	2.93	0.09

Note:

- 1) The figures in the parentheses are the corresponding standard deviations.
- 2) The Likelihood Ratio test is testing $H_0: \beta=1$ against the alternative $H_1: \beta \neq 1$.
- 3) An intercept is included in ((3.1) in the estimation.
- 4) The number of autoregressive lags is selected by minimizing AIC.

From the Table 5 we can see $\beta=1$ cannot be rejected for the US and UK even at the 10% significance level. For Germany and Canada, β is significantly bigger than one at the 1% and 10% level, respectively. Therefore, we have strong evidence supporting the Fisher effect.

3.2 Nonlinearity Tests

We have strong evidence that the nominal interest and inflation are cointegrated, but the path of adjustment to the long-term equilibrium is not necessarily symmetric. If the equilibrium error follows a threshold autoregression (TAR) process, we have a threshold cointegration as described in Balke and Fomby (1997) (see section 1.3.2).

To test for threshold cointegration, we will follow a two-step methodology suggested by the Balke and Fomby (1997). The first-step, which comprises linear cointegration tests, has already been performed in the previous section and cointegration is established for all the countries under study except Italy. The cointegration residuals from the Johansen (1988) procedure are plotted in Figure 6. For the second step, nonlinearity tests including Hansen's (1999) SETAR test, Tsay's (1989) univariate test and Tsay's (1998) multivariate nonlinearity test have been applied to the cointegration residuals. The results are given in Table 6.

Table 6: Nonlinearity Tests on Cointegrating Residual

		Tsay's Univariate F Test		Tsay's Multivariate Test		SETAR(1,2) Test	
		<i>F</i> stat	<i>P</i> value	<i>Cd</i> stat	<i>P</i> value	<i>SupF</i> stat	<i>P</i> value
Johansen	US	1.57	0.21	13.52**	0.04	27.92**	0.02
	UK	1.51	0.19	28.31***	0.01	18.43**	0.05
	Germany	1.73	0.14	10.64*	0.10	62.77**	0.02
	Canada	1.43	0.24	8.04	0.24	30.10***	0.01
Phillips and Ouliaris	US	1.10	0.34	10.94*	0.09	27.95**	0.03
	UK	1.76	0.12	27.14***	0.01	18.44*	0.06
	Germany	2.19**	0.06	7.96	0.24	64.48**	0.02
	Canada	1.63	0.20	6.35***	0.38	23.93**	0.02

*: significant at 10% **: significant at 5% ***: significant at 1%

Note:

- 1) For category “Johansen” and “Phillips and Ouliaris”, the cointegrating vector comes from Johansen (1988) and Phillips-Ouliaris (1990) cointegration procedure, respectively.
- 2) For all three nonlinearity tests, the null hypothesis is that the model is linear and the autoregressive lags are selected by minimizing AIC.
- 3) The Hansen’s SETAR(1,2) is testing one versus two regimes and the *P* values are from Monte Carlo simulation with 2,000 repetitions.

The null hypothesis of linearity is rejected at the 5% level by Tsay’s multivariate test and Hansen’s SETAR(1,2) test in most of the cases. Linearity, however, is not rejected by Tsay’s univariate *F* test, which may result from its low power (see Balke and Fomby (1997)).

The rejection of linearity by Tsay's multivariate test and Hansen's SETAR(1,2) test, however, is not at all surprising because it has been well documented that some fundamental series in the economy exhibit asymmetric adjustment. For example, Neftci (1984) has brought to attention on nonlinearity in the dynamics of US unemployment rates; Hamilton (1989) finds asymmetry in the path of US GNP and models it with a Markov regime-switching process; Hess and Iwata (1997) provide evidences for the presence of nonlinearity in the GDP of G7 countries.

Because the nominal interest rate and inflation are closely related to the unemployment rate, GDP and other fundamental variables in the economy, it is reasonable to suspect that the equilibrium relationship between the nominal interest rate and inflation may exhibit some nonlinear behavior. As a matter of fact, Kesriyeli, et al (2004) have discovered possible asymmetries in the short-term interest rate response to the output gap and inflation, based on the US, UK and Germany data since the early 1980s. If the nominal interest rate and the inflation rate follow an asymmetric path of adjustment to some long-term equilibrium, inferences based on linear cointegration analysis may be misleading. Therefore, it will be of interest to model the Fisher relationship with nonlinear cointegration models. In the next chapter, we will use the threshold error correction model of Hansen and Seo (2002) to examine the Fisher effect.

3.3 End-of-chapter Summary

According to the Johansen (1988) and Phillips-Ouliaris (1990) cointegration tests, the null of no cointegration between the nominal interest rate and the inflation rate is rejected for all the countries under our study except Italy. For Italy, the failure to reject no cointegration may result from its small sample size, and we will exclude Italy from our study in the rest of this dissertation. For all the other countries, linear cointegration analyses seem to support the Fisher effect.

Linear cointegration models, however, may suffer from power distortion in the presence of nonlinearity. As a matter of fact, linearity is rejected by Hansen's SETAR(1,2) and Tsay's multivariate test of nonlinearity in most of the cases. To overcome the limitation of linear models, Hansen and Seo's (2002) two-regime threshold error correction model will be fitted and a comparison of forecast efficiency between linear and nonlinear cointegration analyses will be preformed in the next chapter.

CHAPTER FOUR: THRESHOLD ERROR CORRECTION MODEL

4.1 Hansen and Seo's (2002) Two-Regime TVECM

Linear cointegration analyses in Chapter Three support the existence of a Fisher effect, but the path of adjustment to the long-term equilibrium seems to be nonlinear. Linear cointegration models, including Johansen (1998) and Phillips-Ouliaris (1990), suffer from misspecification if nonlinearity is present.

To model possible nonlinearity in the Fisher relationship, we will go beyond linear cointegration models. The two-regime threshold error correction model (TVECM) proposed by Hansen and Seo (2002), which is nonlinear in nature, is a natural extension of linear cointegration analysis. In their framework, the cointegrating vector is constant across different regimes. The threshold variable is the deviation from the equilibrium in the prior period. To formally set up the model, first define

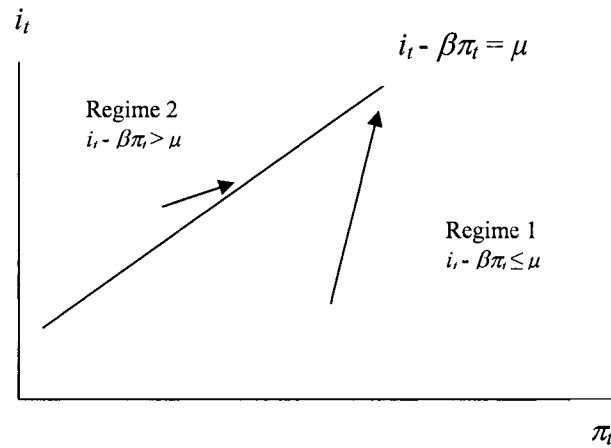
$$\Delta x_t = \begin{bmatrix} \Delta i_t \\ \Delta \pi_t \end{bmatrix}$$

and consider the following two-regime vector threshold error correction model:

$$\Delta x_t = \begin{cases} \begin{pmatrix} a_1^{(1)} \\ a_2^{(1)} \end{pmatrix} + \begin{pmatrix} \theta_1^{(1)} \\ \theta_2^{(1)} \end{pmatrix} (i_{t-1} - \beta\pi_{t-1}) + \sum_{i=1}^p \Psi_i^{(1)} \Delta x_{t-i} + \varepsilon_t & \text{if } (i_{t-1} - \beta\pi_{t-1}) \leq \gamma \\ \begin{pmatrix} a_1^{(2)} \\ a_2^{(2)} \end{pmatrix} + \begin{pmatrix} \theta_1^{(2)} \\ \theta_2^{(2)} \end{pmatrix} (i_{t-1} - \beta\pi_{t-1}) + \sum_{i=1}^p \Psi_i^{(2)} \Delta x_{t-i} + \varepsilon_t & \text{if } (i_{t-1} - \beta\pi_{t-1}) > \gamma \end{cases} \quad (4.1)$$

where $\varepsilon_t \sim N(0, \Sigma)$ and is serially uncorrelated. This model allows asymmetrical adjustment to the long-term equilibrium, as illustrated in Figure 7. The system may exhibit a higher speed of mean reversion in one of the regimes than in the other.

Figure 7: Asymmetric Adjustment



To estimate the model defined in (4.1), Hansen and Seo propose a maximum likelihood methodology. Let

$$A_1 = \begin{bmatrix} a^{(1)} & \theta^{(1)} & \Psi_1^{(1)} & \Psi_2^{(1)} & \dots & \Psi_p^{(1)} \end{bmatrix}$$

$$A_2 = \begin{bmatrix} a^{(2)} & \theta^{(2)} & \Psi_1^{(2)} & \Psi_2^{(2)} & \dots & \Psi_p^{(2)} \end{bmatrix}$$

$$X_{t-1}(\beta) = \begin{pmatrix} 1 \\ z_{t-1}(\beta) \\ \Delta x_{t-1} \\ \Delta x_{t-1} \\ \vdots \\ \Delta x_{t-p} \end{pmatrix}$$

where

$$z_{t-1} = i_{t-1} - \beta \pi_{t-1}$$

$$a^{(1)} = \begin{bmatrix} a_1^{(1)} \\ a_2^{(1)} \end{bmatrix} \quad a^{(2)} = \begin{bmatrix} a_1^{(2)} \\ a_2^{(2)} \end{bmatrix}$$

$$\theta^{(1)} = \begin{bmatrix} \theta_1^{(1)} \\ \theta_2^{(1)} \end{bmatrix} \quad \theta^{(2)} = \begin{bmatrix} \theta_1^{(2)} \\ \theta_2^{(2)} \end{bmatrix}.$$

Then, (4.1) can be compactly written as

$$\Delta x_t = \begin{cases} A_1' X_{t-1}(\beta) + \varepsilon_t & \text{if } z_{t-1} \leq \gamma \\ A_2' X_{t-1}(\beta) + \varepsilon_t & \text{if } z_{t-1} > \gamma \end{cases}$$

or

$$\Delta x_t = A_1' X_{t-1}(\beta) d_{1t}(\beta, \gamma) + A_2' X_{t-1}(\beta) d_{2t}(\beta, \gamma) + \varepsilon_t \quad (4.2)$$

where

$$d_{1t}(\beta, \gamma) = I(z_{t-1}(\beta) \leq \gamma),$$

$$d_{2t}(\beta, \gamma) = I(z_{t-1}(\beta) > \gamma),$$

and $I(\cdot)$ is an indicator function. The threshold effect only has content if

$$0 < \Pr(z_{t-1}(\beta) < \gamma) < 1,$$

because otherwise the model simplifies to linear cointegration. Hansen and Seo assume that

$$\pi_0 \leq \Pr(z_{t-1}(\beta) \leq \gamma) \leq 1 - \pi_0 \quad (4.3)$$

where $\pi_0 > 0$. Under the assumption that $\varepsilon_t \sim \text{i.i.d. } N(0, \Sigma)$, the likelihood function is

$$L_n(A_1, A_2, \Sigma, \beta, \gamma) = -\frac{1}{2} \log(|\Sigma|) - \frac{1}{2} \sum_{t=1}^n \varepsilon_t(A_1, A_2, \beta, \gamma) \Sigma^{-1} \varepsilon_t(A_1, A_2, \beta, \gamma)',$$

where

$$\varepsilon_t(A_1, A_2, \beta, \gamma) = \Delta x_t - A_1' X_{t-1}(\beta) d_{1t}(\beta, \gamma) + A_2' X_{t-1}(\beta) d_{2t}(\beta, \gamma)$$

First, given (β, γ) , we can get the maximum likelihood estimator (MLE) of $\hat{A}_1(\beta, \gamma)$ and $\hat{A}_2(\beta, \gamma)$, which turns out to be the OLS estimator of (4.2) in this case. The estimates for the residuals and the covariance matrix are

$$\begin{aligned} \hat{\varepsilon}_t(\beta, \gamma) &= \hat{\varepsilon}_t(\hat{A}_1, \hat{A}_2, \beta, \gamma) \\ \hat{\Sigma}(\beta, \gamma) &= \frac{1}{n} \sum_{t=1}^n \hat{\varepsilon}_t(\beta, \gamma) \hat{\varepsilon}_t(\beta, \gamma)' \end{aligned}$$

Then the concentrated likelihood function is

$$\begin{aligned} L_n(\beta, \gamma) &= L_n(\hat{A}_1(\beta, \gamma), \hat{A}_2(\beta, \gamma), \hat{\Sigma}(\beta, \gamma), \beta, \gamma) \\ &= -\frac{1}{2} \log|\hat{\Sigma}(\beta, \gamma)| - \frac{np}{2} \end{aligned}$$

The MLE of (β, γ) minimizes $\log|\hat{\Sigma}(\beta, \gamma)|$ subject to the constraint

$$\pi_0 \leq \frac{1}{n} \sum_{t=1}^n I(x_t' \beta \leq \gamma) \leq 1 - \pi_0$$

To test for a threshold, Hansen suggests the SupLM statistic proposed by Davis (1987) to test the null hypothesis $H_0: A_1 = A_2$ against the alternative $H_A: A_1 \neq A_2$. Let $X_1(\beta, \gamma)$ and $X_2(\beta, \gamma)$ be the stacked rows of $X_{t-1}(\beta) d_{1t}(\beta, \gamma)$ and $X_{t-1}(\beta) d_{2t}(\beta, \gamma)$, respectively. Let $\xi_1(\beta, \gamma)$ and

$\xi_2(\beta, \gamma)$ be the matrices of stacked rows of $\hat{\varepsilon}_t(\beta, \gamma) \otimes X_{t-1}(\beta) d_{1t}(\beta, \gamma)$ and $\hat{\varepsilon}_t(\beta, \gamma) \otimes X_{t-1}(\beta) d_{2t}(\beta, \gamma)$, respectively. Define

$$M_1(\beta, \gamma) = I_p X_1(\beta, \gamma)' X_1(\beta, \gamma)$$

$$M_2(\beta, \gamma) = I_p X_2(\beta, \gamma)' X_2(\beta, \gamma)$$

and

$$\Omega_1(\beta, \gamma) = \xi_1(\beta, \gamma)' \xi_1(\beta, \gamma)$$

$$\Omega_2(\beta, \gamma) = \xi_2(\beta, \gamma)' \xi_2(\beta, \gamma).$$

Define $\hat{V}_1(\beta, \gamma)$ and $\hat{V}_2(\beta, \gamma)$, the Eicker-White covariance matrix for $\hat{A}_1(\beta, \gamma)$ and $\hat{A}_2(\beta, \gamma)$, as

$$\hat{V}_1(\beta, \gamma) = M_1(\beta, \gamma)^{-1} \Omega_1(\beta, \gamma) M_1(\beta, \gamma)^{-1}$$

$$\hat{V}_2(\beta, \gamma) = M_2(\beta, \gamma)^{-1} \Omega_2(\beta, \gamma) M_2(\beta, \gamma)^{-1}$$

The standard expression for the heteroskedasticity-robust LM-like statistic is

$$LM(\beta, \gamma) = \text{vec}(\hat{A}_1(\beta, \gamma) - \hat{A}_2(\beta, \gamma))' (\hat{V}_1(\beta, \gamma) - \hat{V}_2(\beta, \gamma))^{-1} \text{vec}(\hat{A}_1(\beta, \gamma) - \hat{A}_2(\beta, \gamma))$$

If (β, γ) were known, the $LM(\beta, \gamma)$ would be the test statistic. When (β, γ) are unknown, we can evaluate $LM(\beta, \gamma)$ at point estimates obtained under H_0 . Suppose the estimate for β is $\hat{\beta}$ under the null. However, there is no estimate for γ under H_0 , so there is no conventionally defined LM statistic. The $\text{Sup}LM$ is defined as

$$\text{Sup}LM = \sup_{\gamma_L \leq \gamma \leq \gamma_U} LM(\tilde{\beta}, \gamma) \quad (4.4)$$

where $\tilde{\beta}$ is the estimate of β under the null of one regime. For this test, the search region $[\gamma_L \ \gamma_U]$ is set so that the γ_L is the π_0 percentile of $z_{t-1}(\hat{\beta})$, and γ_U is the $(1 - \pi_0)$ percentile. The $\text{Sup}LM$ statistic follows a nonstandard distribution. Hansen and Seo suggest using bootstrap methods to get critical values.

If the relationship between the nominal interest and inflation rate can be well described by the two-regime TVECM model and the Fisher effect exists in the long run, β should be equal to one when there is no tax effect and bigger than one when there is a tax effect.

4.2 The Estimated Threshold Error Correction Model

The two-regime TVECM model as defined in (4.1) has been used to model the relationship between the nominal interest and inflation rate of the United States, the United Kingdom, Germany and Canada. Details about the data can be found in Section 1.4. The maximum likelihood procedure of Hansen and Seo (2002) has been implemented in the estimation. Part of the results is presented in Table 7.

Table 7: Estimation of Fisher Effect with Threshold Error Correction Model

Country	From Johansen Procedure	From Threshold Error Correction Model ¹				
	β	β	Estimated Threshold	SupLM	Critical Values for SupLM	
					10%	5%
US	1.27	1.20	-3.45%	10.57	17.42	18.61
UK	0.90	1.64	-2.19%	17.34*	16.90	18.16
Germany	1.59	1.11	5.26%	12.53	16.37	18.04
Canada	1.38	1.22	2.54%	14.09	22.83	24.15

*: significant at 10% **: significant at 5% ***: significant at 1%

Note:

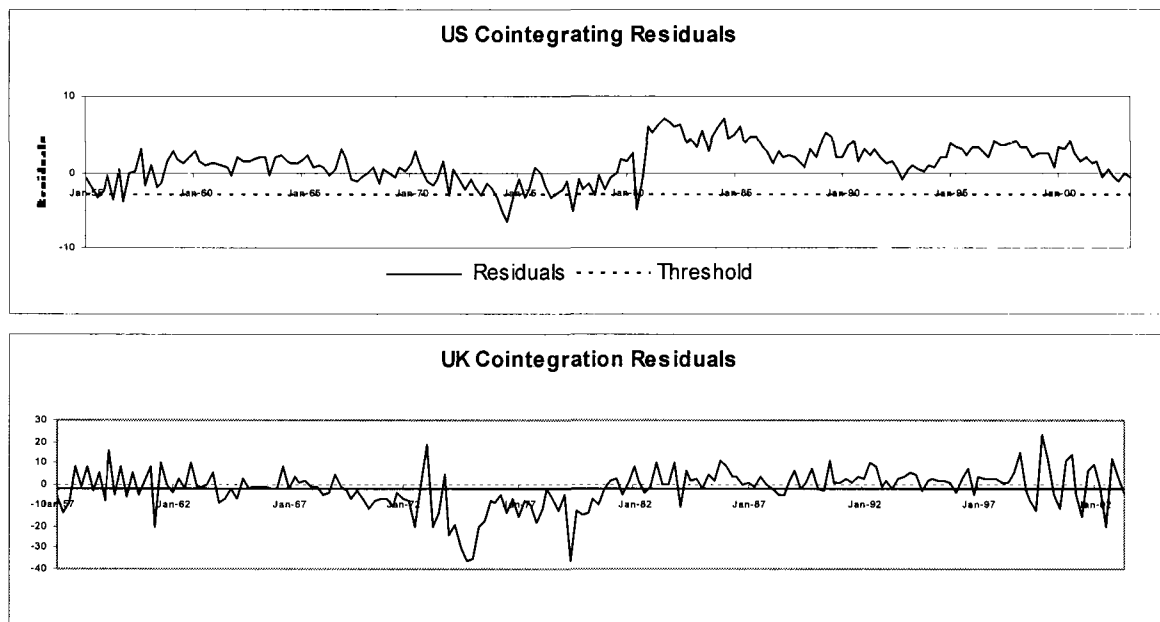
- 1) The threshold error correction is set up as in Equation (4.1) and the number of autoregressive lags (p) is selected by minimizing AIC.
- 2) The SupLM test the null hypothesis of one regime against the alternative hypothesis of two regimes. Under the null, the model is linear.
- 3) For all the countries, π_0 (as in (4.3)) is set to 5% to ensure that there are at least 5% of observations in each regime. The results are not quite sensitive to the choice of π_0 .
- 4) The critical values come from Monte Carlo simulation with 2000 replications.
- 5) The estimates of β from Johansen procedure are listed here for comparison.

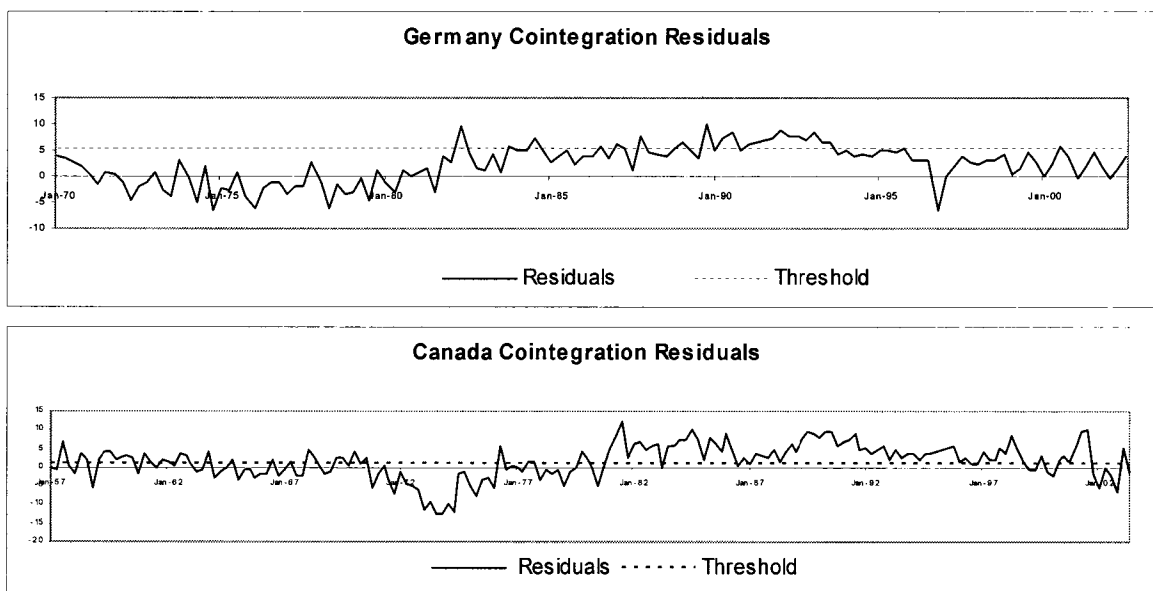
¹ For the countries under study, the number of observations in the upper regime (above the threshold) and lower regime (below threshold) can be found in the following table.

<u>Number of Observations</u>	<u>US</u>	<u>UK</u>	<u>Germany</u>	<u>Canada</u>
Above Threshold	181	103	88	83
Below Threshold	13	83	24	103

For all the countries under our study, the TVECM estimates of β are greater than one, which is consistent with a Fisher effect accounting for taxes. The SupLM test, however, fails to reject the null hypothesis of linearity except for a weak rejection at the 10% level in UK. This may result from the limited sample size of our quarterly dataset since TVECM estimation typically requires a large sample size. The estimated threshold γ varies across countries. The equilibrium errors, together with the estimated threshold, are plotted in Figure 8.

Figure 8: The Estimated Cointegration Residuals and Threshold from TVECM





Since the estimated thresholds vary across different countries in our study, we may be interested in the interpretation of the threshold. First, recall the Fisher equation defined in (1.1),

$$i_t = E_t r_t + E_t \pi_t.$$

Rearrange and we can get

$$i_t - E_t \pi_t = E_t r_t. \quad (4.5)$$

As we have discussed in Section 1.1, investors consider assets with yields that are in real terms, such as physical capital, as very close substitutes for bonds, a class of assets whose returns are in nominal terms. In (4.5), the left side, $i_t - E_t \pi_t$, is the inflation adjusted yield on bonds. The right side, $E_t r_t$, is the expected yield in the real sector. In equilibrium, people will ask for the same rate of return and (4.5) is the result of the law of one price.

The analysis above, however, assumes that the investors possess perfect information about the exact yields in the bond market and the real sector, and that transferring investment between them is costless. In the real world, investors may have less than full information about the yield rates, and the redirection of investments from one market to the other is not costless. For example, to move investment from the bond market to production, we should consider the cost of selling the bonds and the costs of entry into the real sector, including the expenditures associated with finding profitable projects, consulting fee, project evaluation fee, etc. On the other hand, when there is uncertainty associated with the yield in the bond market and/or production sector, most of the investors will just wait until the yield gap is wide enough to justify the reinvestment. Correspondingly, we may have two regimes when it comes to the speed of adjustment to the long-term equilibrium. For example, in the United States, if the inflation adjusted yield in the bond market is below the threshold (with an estimate of -3.45%), investors will have a sure sign that transferring funds to the real sector is more profitable, even after paying possible expenses associated with the reinvestment. Therefore, capital will flow faster from the bond market to the real sector, compared with the relatively lower adjustment speed in the other regime. At the same time, the threshold itself may depend on a lot of factors in the economy, including the yield in the real sector, the maturity of the financial market, the cost of transferring investment between the bond market and the real sector, etc. These factors could be very different in different economies, which may lead to varied threshold values across countries.

Given our estimated two-regime TVECM, we might be tempted to perform inferences on the Fisher effect β and the threshold γ . However, within in the threshold cointegration setting, there is no formal theory on the asymptotic properties of these two parameters, according to Hansen and Seo (2002). Therefore, at this stage, we still can not test the type of hypotheses like $\beta = 1$ and $\gamma = 0$. It may be another interesting area for future research.

4.3 Test of Equal Forecast Efficiency

The Fisher effect has been estimated with linear and nonlinear cointegration models. The next step is to compare their performance. The comparison, of course, can be done in many different directions. One intuitive way is to look at the forecast accuracy of the two classes of models. We are going to compare one-step-ahead forecast accuracy of the Johansen linear cointegration model and the two-regime TVECM following the methodology described in Clark and McCracken (2001).

4.3.1 Test of Encompassing

As is well known, the Johansen linear cointegration model is nested within the two-regime TVECM. Therefore, we are going to adopt the Clark and McCracken (2001) test of encompassing, which is appropriate when comparing the forecast efficiency of two nested models.

To generate one-step-ahead forecast errors, we divide the sample into in-sample and out-of-sample portions as in Clark and McCracken (2001). Let R and P stand for the number of observations in the in-sample and out-of-sample portions, respectively, so that the total sample size $T=R+P$. The forecasts are one-step-ahead, recursive post-sample predictions for the 3-month inflation rate based on Johansen (1998) and the two-regime TVECM. First, we use the beginning R observations (the in-sample portion of the sample) for estimation. Then, based on the estimated model, we compute the one-step-ahead inflation forecast and the corresponding forecast error in period $(R+1)$. After that, the first $(R+1)$ observations are used for estimation and the one-step-ahead forecast error for inflation in period $(R+2)$ are derived. We continue with the process until the end of the sample, period $R+P$, is reached.

To test the null hypothesis of equal forecast efficiency, we are going to look at two test statistics applied in Clark and McCracken (2001): ENC-T and ENC-NEW. The ENC-T test statistic was first proposed by Harvey et al. (1998). In our case, we have two series of one-step-ahead inflation forecasts from the linear cointegration model and the two-regime TVECM, denoted by f_{1t} and f_{2t} respectively. Consider a linear composite prediction of the form

$$\pi_t = \lambda_1 f_{1t} + \lambda_2 f_{2t} + \varepsilon_t \quad (4.6)$$

where π_t is the inflation rate in period t , λ_1 and λ_2 are fixed coefficients, and ε_t is the composite prediction error. The linear composite prediction in (4.6) is studied in Nelson (1972). We can use least squares regression to get estimates for λ_1 and λ_2 that minimize

$\sum \hat{\varepsilon}_t^2$. For (4.6), the least square regression provides the minimum mean square error linear composite prediction for the sample period. Under the assumption that both f_{1t} and f_{2t} are individually unbiased, (4.6) can be rewritten as¹

$$\pi_t = (1 - \lambda) f_{1t} + \lambda f_{2t} + \varepsilon_t. \quad (4.7)$$

or

$$\pi_t - f_{1t} = \lambda (f_{2t} - f_{1t}) + \varepsilon_t. \quad (4.8)$$

As we can see from (4.8), the greater the ability of the forecast difference ($f_{2t} - f_{1t}$) to explain the Johansen forecast error ($\pi_t - f_{1t}$), the larger will be the weights given to the TVECM forecast f_{2t} . In the case $\lambda = 0$, f_{1t} is “encompassing” f_{2t} in the sense that f_{1t} contains all the information present in f_{2t} , i.e., the TVECM predictions contain no more information than that already incorporated in the linear cointegration model predictions.

Define the forecast errors

$$e_{1t} = (\pi_t - f_{1t}), \quad (4.9)$$

$$e_{2t} = (\pi_t - f_{2t}). \quad (4.10)$$

Substitute (4.9) and (4.10) into (4.7) and after rearrangement we can get

$$e_{1t} = \lambda (e_{1t} - e_{2t}) + \varepsilon_t. \quad (4.11)$$

¹ See the Nelson (1972).

To test the null hypothesis $\lambda = 0$, Harvey et al. (1998) propose using a t -like statistic for the covariance between e_{1t} and $(e_{1t} - e_{2t})$. Denote $d_t = e_{1t}(e_{1t} - e_{2t})$ and $\bar{d} = P^{-1} \sum d_t$, the ENC-T test statistic is defined as

$$\text{ENC-T} = (P-1)^{1/2} \frac{\bar{d}}{\sqrt{P^{-1} \sum_t (d_t - \bar{d})^2}} . \quad (4.12)$$

In the ENC-T test statistic, \bar{d} (the covariance between e_{1t} and $e_{1t} - e_{2t}$) is scaled by an estimate of the standard deviation of \bar{d} . Clark and McCracken (2001) propose using the variance of one of the forecast errors, and the new test statistic is defined as

$$\text{ENC-NEW} = P \frac{\bar{d}}{MSE_2} = P \frac{P^{-1} \sum_t e_{1t} (e_{1t} - e_{2t})}{P^{-1} \sum_t e_{2t}^2} . \quad (4.13)$$

For both tests, under the null the linear cointegration model forecast encompasses TVECM, the covariance between e_{1t} and $(e_{1t} - e_{2t})$ will be less than or equal to zero. Under the alternative that TVECM contains added information, the covariance should be positive. Therefore, both ENC-T and ENC-NEW test are one-sided. Clark and McCracken (2001) show that under some regularity conditions, the asymptotic distribution of both ENC-T and ENC-NEW test statistic will converge to nonstandard distributions if $P/R \rightarrow \pi > 0$. The limiting distributions depend on the number of restrictions on the parameters¹ and π . Clark and McCracken suggest using bootstrap procedures to get the critical values.

¹ In the Clark and MaCracken (2001), they consider two nested models. In this dissertation, the linear cointegration model is nested within the two-regime TVECM. The number of restrictions refers to the number of restrictions put on the TVECM to get the linear cointegration model.

4.3.2 Empirical Results

Based on the recursive one-step-ahead inflation forecast from Johansen's (1988) linear cointegration model and the TVECM, the ENC-T and ENC-NEW statistics have been computed. To get the bootstrap critical values, we are following the following steps:

1. With the whole sample, estimate the Johansen linear cointegration model with lags determined by minimizing AIC. Next, take a simple random sample with replacement from the residuals and use the estimated parameters to simulate the dynamics of the 3-month T-Bill rate and inflation. The initial values are the same as those in the real data.
2. Estimate the Johansen linear model and TVECM model, compute the out-of-sample forecast errors based on the simulated data, and calculate the ENC-T and ENC-NEW test statistic¹;
3. Repeat the process with 2,000 replications and compute the 95% and 99% critical values.

Based on the 3-month T-Bill rate and inflation from the United States, the United Kingdom, Germany and Canada, the computed ENC-T and ENC-NEW test statistics are presented in Table 8, together with their bootstrap critical values.

¹ Here we follow the same procedure that has been used to compute the recursive one-step-ahead forecast errors described in Section 4.3.1.

Table 8: Test of Encompassing for Equal Forecast Efficiency

		Test Stat	Distribution of Test Statistics (From Bootstrap)							
			0.5%	2.5%	5%	10%	90%	95%	97.5%	99.5%
ENC-T	US	-0.64	-2.91	-1.96	-1.66	-1.43	0.84	1.13	1.57	2.51
	UK	1.02	-2.33	-2.17	-1.94	-1.55	1.41	1.85	2.10	2.66
	Germany	2.51 ***	-2.47	-2.33	-1.69	-1.35	1.28	1.60	1.88	2.01
	Canada	-1.55	-2.87	-1.94	-1.68	-1.25	0.98	1.05	1.12	1.21
ENC-NEW	US	-1.64	-6.79	-3.72	-3.07	-2.69	1.59	3.48	5.88	12.94
	UK	9.61 **	-8.31	-4.01	-3.71	-2.67	4.10	5.10	6.28	10.91
	Germany	7.57 **	-3.75	-2.85	-2.73	-1.94	2.91	4.29	8.91	23.54
	Canada	-3.33	-5.57	-3.94	-3.28	-2.23	1.99	2.76	3.09	3.40

*: significant at 10% **: significant at 5% ***: significant at 1%

Notes:

1. Both ENC-T and ENC-NEW tests are one-sided.
2. Under the null, the linear cointegration model encompasses the TVECM and both statistics are equal to or less than zero.
3. Under the alternative, the TVECM contains added information and the test statistics are positive.

The ENC-T test statistic for Germany is significant at 1% level, which implies that the nonlinear TVECM is better in the sense that it contains more information than that in the linear model. Likewise, the ENC-NEW test statistic is significant at the 5% level for the United Kingdom and Germany. But none of the test statistics are significant for US. Therefore, we have mixed results for the null hypothesis of equal forecast efficiency.

4.4 End-of-Chapter Summary

With Hansen and Seo's (2002) two-regime threshold error correction model, we have found further evidence of the Fisher effect in the United States, the United Kingdom, Germany and Canada. The Sup LM test of regimes fails to reject the null hypothesis of linearity though.

To compare the efficiency of linear and nonlinear cointegration models, we have performed the Clark and McCracken (2001) encompassing test and the results are mixed for the null hypothesis of equal forecast efficiency.

CHAPTER FIVE: CONCLUSIONS AND FINAL REMARKS

One of the fundamental hypotheses in the history of economics is the Fisher hypothesis, which assumes that the nominal interest rate changes one-for-one with the inflation rate. However, it is controversial in theory as well as in empirical studies. In this dissertation we have reviewed extensively the existing literature and carried out a series of analyses, including linear and nonlinear cointegration models, to test the existence of a long-run Fisher effect for the United States, the United Kingdom, Germany, Italy and Canada. In both linear and nonlinear cointegration models we have found evidences supporting the Fisher hypothesis.

In Chapter One, we first reviewed the controversy on the Fisher effect in the theoretical literature and the empirical studies, which typically involves linear cointegration analysis in time series. Then, going beyond the linear models, we looked at the literature on nonlinear cointegration analysis including the threshold cointegration and threshold error correction model. The nonlinear models allow asymmetric adjustment to the long-term attractor and provide a more general setting for testing the Fisher effect.

In Chapter Two, we applied the ADF test, Ng-Perron test and Enders and Granger test to check the order of integration for the 3-month T-Bill rate and inflation for the United States, the United Kingdom, Germany, Italy and Canada. There was strong evidence from all three tests that both the nominal interest rate and inflation rate are integrated of order one.

In Chapter Three, we first applied linear cointegration tests, including Johansen (1988) maximum likelihood ratio test and Phillips and Ouliaris (1990) multivariate P_2 test, to the nominal interest rate and the inflation rate. Cointegration was established for all the countries under study except Italy and the results support the existence of the Fisher effect. The path of adjustment to the long-run equilibrium, however, is not necessarily symmetric as described in linear models. To test for possible nonlinearity, we applied a series of nonlinearity tests to the equilibrium errors and linearity was rejected by Tsay's (1998) multivariate test and Hansen's SETAR(1,2) in most of the cases.

With the presence of nonlinearity, threshold cointegration models provide a better description of the long-run equilibrium between nominal interest rate and the inflation rate. Therefore, in Chapter Four, we estimated Hansen and Seo's (2002) two-regime threshold error correction model and found further support of the existence of the Fisher effect in all the countries under study. To compare the efficiency linear and nonlinear models, the ENC-T and ENC-NEW encompassing tests as described in Clark and McCracken (2001) have been applied to compare the post-sample forecast efficiency of Johansen (1988) linear cointegration model and Hansen and Seo's (2002) two-regime TVECM. The results are mixed for the null hypothesis of equal forecast efficiency.

In summary, our study supports the long-run Fisher effect. Still, there exist areas for future research:

- Derive the asymptotic distribution theory for the parameter estimates in threshold error correction model. As pointed out in Chan (1993) and Hansen (2000), the parameter estimates in threshold models follow non-standard distribution, which makes it challenging to work out such a distribution.
- Develop a methodology to exclude a time trend in the two-regime threshold cointegration model and include constant in the equilibrium. Doing that would necessitate imposing constraints on the intercepts, and it is not immediately apparent how to impose. This could be a fruitful area of research, according to Hansen and Seo (2002),

APPENDIX A: TECHNICAL BACKGROUND

Weakly Stationary

In time series, a univariate sequence $\{x_t\}$ is weakly stationary if (i) $E[x_t]=\mu$ for $\mu \in R$; (ii) $\text{cov}(x_t, x_{t+h})=\sigma_h^2$ and σ_h^2 is independent of time t for integers h ; (iii) $\text{var}(x_t)<\infty$.

Intuitively, a weakly stationary process is mean-reverting, that is, it keeps reverting to the common mean dynamically. Thus the effect of any disturbance to a weakly stationary process is temporary and will soon die out. In time series, "weakly stationary" is commonly referred to as "stationary" for convenience.

Unit Root

A process $\{x_t\}$ contains a unit root if $x_t=x_{t-1}+\varepsilon_t$, where ε_t is a stationary process. If a process contains a unit root, it is not stationary and any disturbance to the process will have a permanent effect. A process containing a unit root is integrated of order one, denoted by $I(1)$. Likewise, a process contains d unit roots if d^{th} -order difference of the sequence contains a unit root for $d \geq 2$ and the sequence is $I(d)$. Stationary processes are $I(0)$. Since most economic variables are $I(1)$ or $I(0)$, we are going to focus on $I(1)$ processes.

Cointegration

If two processes, $\{x_t\}$ and $\{y_t\}$, are both $I(1)$ and there is a linear combination, $x_t-\beta y_t$, that is $I(0)$, then these two variables are cointegrated. The interpretation of cointegration is

that there is a long-run dynamic equilibrium between these two variables, and the bivariate system keeps returning to this equilibrium despite disturbances.

Error Correction Model (ECM)

The interesting feature of cointegration is that although the individual variables are nonstationary, there is a long-run equilibrium relationship between them. Therefore, cointegration necessitates that at least the movement of one variable responds to the disequilibrium. An error correction model is a specification in which the short-run dynamics of the variables are influenced by the equilibrium error in the previous period. The following is a bivariate error correction model:

$$\Delta x_t = \theta_1(x_{t-1} - \beta y_{t-1}) + \varepsilon_{xt},$$

$$\Delta y_t = \theta_2(x_{t-1} - \beta y_{t-1}) + \varepsilon_{yt},$$

In this model, both variables respond to the equilibrium error. Engle and Granger (1987) show that for any set of cointegrated variables, an error correction model exists. This is known as the Granger representation theorem. Cointegration and error correction models in a more general setting are discussed in Engle and Granger (1987).

Spurious Regression

Suppose two sequences $\{x_t\}$ and $\{y_t\}$ are both $I(1)$ and independent. Consider the commonly used OLS regression

$$y_t = \alpha + \beta x_t + e_t. \tag{A1}$$

Since these two sequences are independent, the regression is meaningless and $\{x_i\}$ should not be predictive for $\{y_i\}$. One might expect β to be statistically insignificant in the regression. Granger and Newbold (1974) generate many such samples, estimate a regression in the form of (A1) for each sample and find that the null hypothesis of $\beta=0$ cannot be rejected in approximately 75% of total simulations, at the 5% significance level.

Furthermore, the regressions usually have a high R^2 and the estimated residuals are highly correlated. This is what Granger and Newbold call “spurious regression”, in which the regression output looks good but the results are meaningless in economics. The cause of spurious regressions is that the residuals sequence $\{e_t\}$ contains a unit root and its variance approaches infinity as t increases, violating some of the fundamental assumptions in the classical regression. As a result, the commonly used asymptotic theory for classical OLS regression does not hold.

APPENDIX B: TABLES

**Table B1: ADF Test of the Nominal Interest Rate, Inflation
And Real Interest Rate**

Country	Sample Size	ADF (no intercept or trend)		ADF (with intercept, no trend)	
		Lags	τ	Lags	τ_{μ}
Nominal Interest Rate					
US	194	2	-0.91	2	-2.04
UK	186	3	-0.98	1	-2.65*
Germany	112	1	-1.08	1	-1.97
Italy	106	2	-1.03	2	-0.51
Canada	186	2	-0.96	1	-2.43
Inflation Rate					
US	194	3	-1.07	3	-1.60
UK	186	3	-1.65*	3	-2.54
Germany	112	3	-1.55	2	-2.86*
Italy	106	2	-1.16	2	-1.45
Canada	186	2	-1.47	2	-2.62*
Real Interest Rate (no tax effect)					
US	194	3	-1.85*	1	-3.55***
UK	186	3	-2.94***	3	-3.25**
Germany	112	3	-1.37	1	-5.44***
Italy	106	2	-1.53	1	-3.05**
Canada	186	2	-2.45**	2	-2.90**

*: significant at 10% **: significant at 5% ***: significant at 1%

Note:

1) For the category “Without Intercept or Time Trend”, the regression equation is

$$\Delta y_t = \gamma y_{t-1} + \sum_{i=1}^p \beta_i \Delta y_{t-i} + \varepsilon_t \quad (\text{a});$$

For “With Intercept, No Time Trend”, the regression is

$$\Delta y_t = \mu + \gamma y_{t-1} + \sum_{i=1}^p \beta_i \Delta y_{t-i} + \varepsilon_t \quad (\text{b}).$$

- 2) The statistics labeled as τ and τ_{μ} are the corresponding statistics to use for equations (a) and (b), respectively.
- 3) The number of lags (p in equation (a) and (b)) for the ADF test is selected by minimizing AIC.
- 4) For τ statistic, the critical values are -2.58 (1%), -1.95(5%) and -1.62(10%); for τ_{μ} statistic, -3.46 (1%), -2.88(5%) and -2.57(10%).

Table B2: ADF Test of the First-Order Difference of the Nominal Interest Rate, Inflation and Real Interest Rate

Country	Sample Size	ADF (no intercept or trend)		ADF (with intercept, no trend)	
		Lags	τ	Lags	τ_{μ}
Nominal Interest Rate (first-order difference)					
US	193	1	-10.54***	1	-10.51***
UK	185	3	-6.88***	3	-6.86 ***
Germany	130	1	-5.44***	1	-5.44 ***
Italy	111	1	-6.42***	1	-6.48 ***
Canada	185	1	-9.75***	1	-9.72 ***
Inflation Rate (first-order difference)					
US	193	2	-12.19***	2	-12.16 ***
UK	185	6	-6.39***	6	-6.37 ***
Germany	130	2	-10.96***	2	-10.92 ***
Italy	111	1	-12.15***	1	-12.10 ***
Canada	185	1	-17.48***	1	-17.43 ***
Real Interest Rate (first-order difference)					
US	193	2	-11.88***	2	-11.85 ***
UK	185	2	-14.89***	2	-14.85 ***
Germany	130	2	-10.34***	2	-10.29 ***
Italy	111	1	-12.20***	1	-12.14 ***
Canada	185	1	-15.44 ***	1	-15.39 ***

*: significant at 10% **: significant at 5% ***: significant at 1%

Note:

- 1) For the category “Without Intercept or Time Trend”, the regression equation is

$$\Delta y_t = \gamma y_{t-1} + \sum_{i=1}^p \beta_i \Delta y_{t-i} + \varepsilon_t \quad (\text{a});$$

For “With Intercept, No Time Trend”, the regression is

$$\Delta y_t = \mu + \gamma y_{t-1} + \sum_{i=1}^p \beta_i \Delta y_{t-i} + \varepsilon_t \quad (\text{b}).$$

- 2) The statistics labeled as τ and τ_μ are the corresponding statistics to use for equations (a) and (b), respectively.
- 3) The number of lags (p in equation (a) and (b)) for the ADF test is selected by minimizing AIC.
- 4) For τ statistic, the critical values are -2.58 (1%), -1.95(5%) and -1.62(10%); for τ_μ statistic, -3.46 (1%), -2.88(5%) and -2.57(10%).

**Table B3: ADF^{GLS} Unit Root Test on Nominal Interest rate,
Inflation and Real Interest Rate**

		Nominal Interest Rate		Inflation		Real Interest Rate	
Country	Sample Size	Lags	Test Statistic	Lags	Test Statistic	Lags	Test Statistic
US	194	1	-1.35	3	-1.40	7	-1.44
UK	186	1	-1.91*	8	-1.87*	8	-1.64*
Germany	112	1	-1.98**	3	-0.72	3	-1.11
Italy	106	1	-0.94	5	-1.15	2	-1.10
Canada	186	2	-1.60	2	-2.37**	6	-1.90*

*: significant at 10% **: significant at 5% ***: significant at 1%

Note:

- 1) The test statistic is from the equation $\Delta y_t = \gamma_{t-1} + \sum_{i=1}^p \beta_i \Delta y_{t-i} + \varepsilon_t$, where y_t is the GLS detrended (by an intercept) series and p is selected minimizing Modified Information Criteria as in Ng and Perron (2001).
- 2) The critical values are -2.58 (1%), -1.95(5%) and -1.62(10%).

Table B5: Johansen (1988) Cointegration Test

		AIC				
Country	Sample Size	lag=1	lag=2	lag=3	lag=4	lag=5
US	194	<u>13.25</u>	18.27	19.57	21.71	26.87
UK	186	<u>298.90</u>	299.58	296.25	298.06	299.27
Germany	112	<u>13.02</u>	17.59	21.47	25.69	33.77
Italy	106	100.68	<u>91.59</u>	96.65	100.87	100.93
Canada	186	168.93	<u>157.96</u>	164.04	167.94	173.64
		Estimated Fisher Effect (β)				
Country	Sample Size	lag=1	lag=2	lag=3	lag=4	lag=5
US	194	<u>-1.27</u>	-1.27	-0.94	-0.78	-0.44
UK	186	<u>-0.90</u>	-0.86	-0.81	-1.27	-1.51
Germ	112	<u>-1.59</u>	-1.46	-1.38	-1.13	-1.12
Ital	106	-1.33	<u>-1.24</u>	-1.05	-1.29	-1.04
Cana	186	-1.68	<u>-1.38</u>	-1.26	-2.94	-2.00
		$\lambda_{\text{trace}}(0)$				
Country	Sample Size	lag=1	lag=2	lag=3	lag=4	lag=5
US	194	<u>23.61</u>	20.60	19.63	15.84	14.77
UK	186	<u>47.43</u>	27.20	17.08	17.78	21.36
Germany	112	<u>42.18</u>	22.31	17.96	17.14	17.42
Italy	106	13.46	<u>13.38</u>	13.59	16.13	14.24
Canada	186	32.72	<u>24.20</u>	19.84	17.95	15.81

		$\lambda_{\max}(0,1)$				
Country	Sample Size	lag=1	lag=2	lag=3	lag=4	lag=5
US	194	<u>18.77</u>	15.58	13.74	8.61	8.85
UK	186	<u>41.05</u>	21.35	12.24	10.73	14.17
Germany	112	<u>38.27</u>	18.49	14.46	11.73	11.90
Italy	106	12.68	<u>10.74</u>	9.91	10.63	10.62
Canada	186	26.32	<u>17.62</u>	12.67	10.82	8.33

Note:

- 1) An intercept is included in the cointegration procedure ((3.1)).
- 2) The number of autoregressive lags is selected by minimizing AIC, and the corresponding statistics are underscored.
- 3) The null hypothesis for both $\lambda_{\text{trace}}(0)$ and $\lambda_{\max}(0,1)$ is that there is no cointegration.

Table B6: Phillips-Ouliaris (1990) Multivariate

Pz Cointegration Test

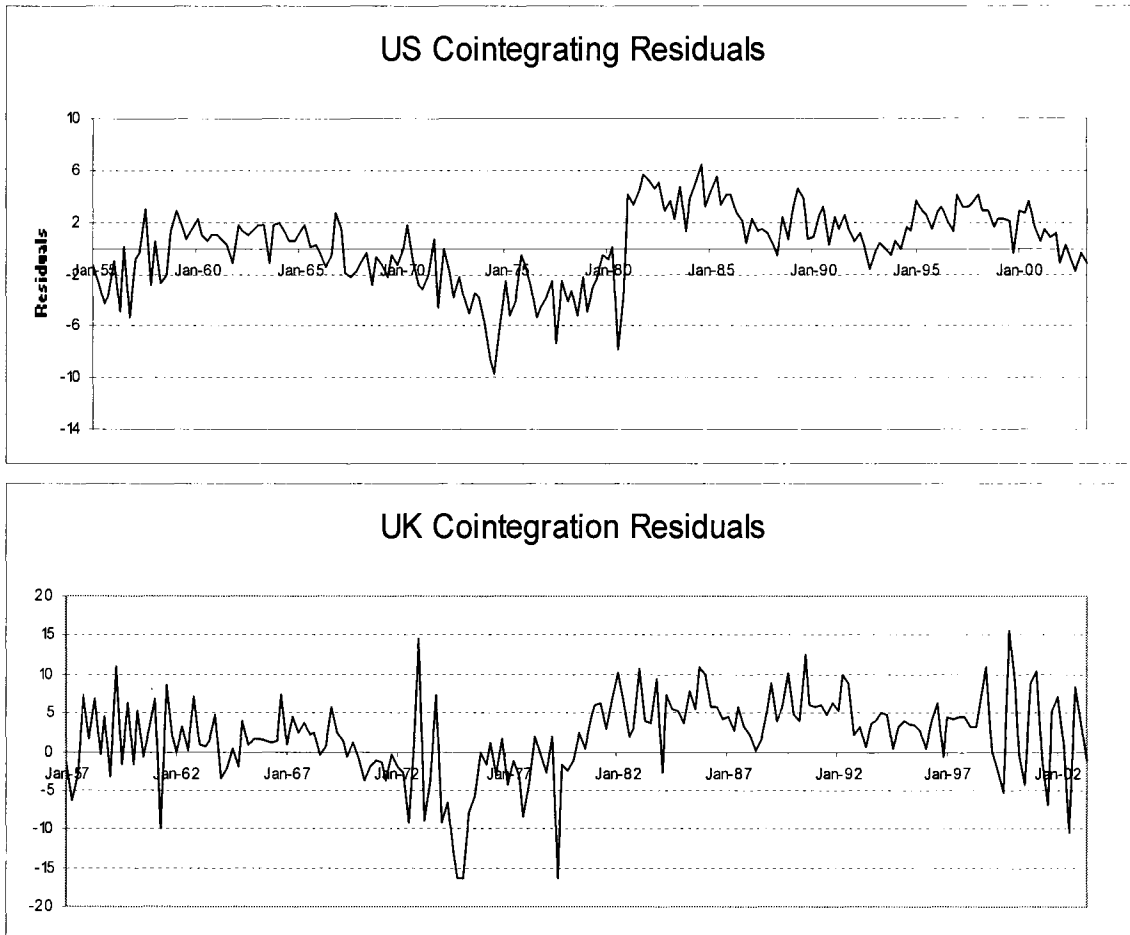
Country	Sample Size	Window Size	Pz	β
US	194	3.61	65.09 **	-1.46
UK	186	0.85	165.68 ***	-0.96
Germany	112	2.83	132.66 ***	-1.77
Italy	106	1.70	46.73	-1.33
Canada	186	2.12	84.53 ***	-1.47

*: significant at 10% **: significant at 5% ***: significant at 1%

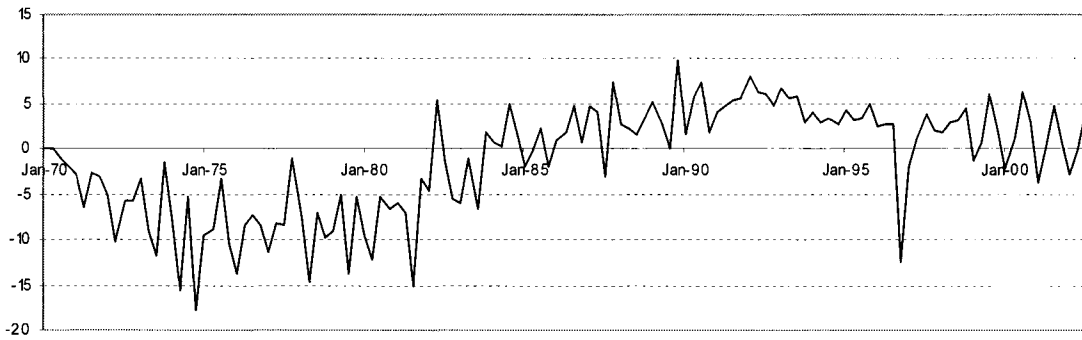
Note:

- 1) An intercept is included in the VAR ((3.2)).

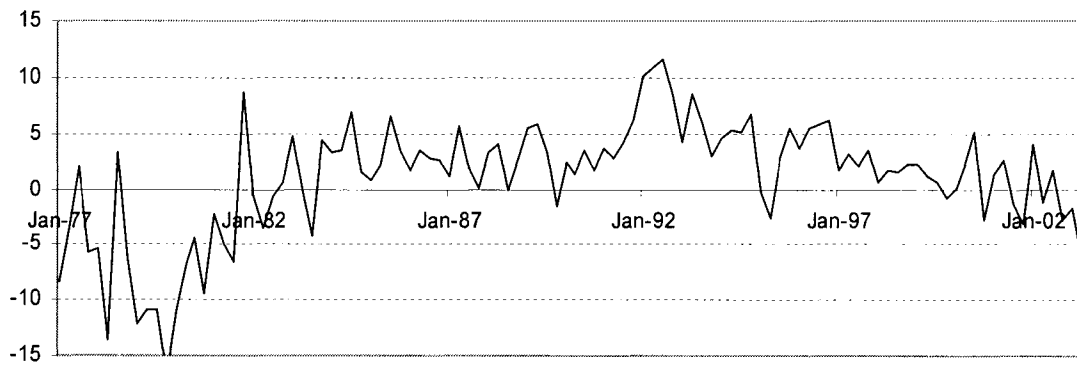
- 2) Automatic window sizes are used as suggested in Andrews (1991).
- 3) The null hypothesis for Pz test is that there is no cointegration.
- 4) The critical values for Pz statistic (demeaned) come from Phillips-Ouliaris (1990):
47.59(10%), 55.22(5%) and 71.93(1%).

APPENDIX C: FIGURES**Figure C1: Plot of Cointegration Residuals from
Phillips-Ouliaris (1990) Procedure**

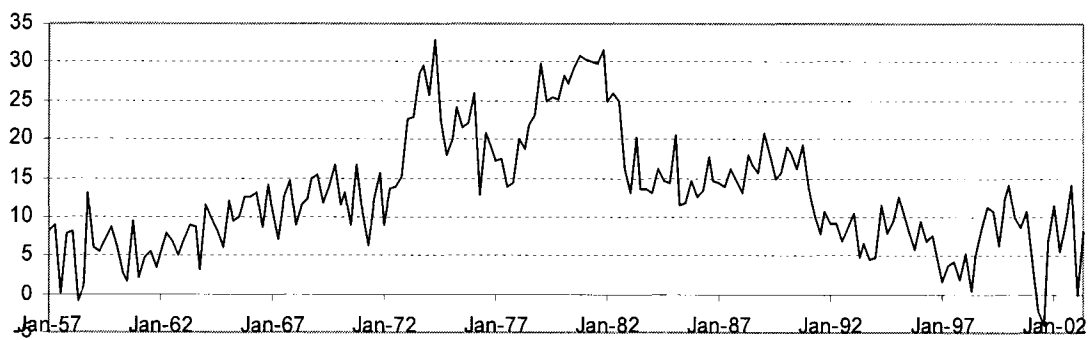
Germany Cointegration Residuals



Italy Cointegration Residuals



Canada Cointegration Residuals



REFERENCES

- Abel, A. (1985): "Dynamic Behavior of Capital Accumulation in a Cash-in-Advance Model," *Journal of Monetary Economics*, v16, 55-71.
- Ahmed, S. and J. H. Rogers (2000): "Inflation and the Great Ratios: Long Term Evidence from the US," *Journal of Monetary Economics*, v45, 3-36.
- Andrews, D. W. K. (1991): "Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimation," *Econometrica*, v59, 817-858.
- Balke, N. S. and T. B. Fomby (1997): "Threshold Cointegration," *International Economic Review*, v38, 627-645.
- Chan, K. S. (1993): "Consistency and Limiting Distribution of the Least Squares Estimator of the Threshold Autoregressive Model," *Annals of Statistics*, v21, 520-533.
- Clark, T. E. and McCracken M. W. (2001): "Tests of Equal Forecast Accuracy and Encompassing for Nested Models", *Journal of Econometrics*, v105, 85–110.
- Cooley, T. F. and G. Hansen (1989): "The Inflation Tax in a Real Business Cycle Model," *The American Economic Review*, v79, 733-748.
- Crowder, W. J. (1997): "The Long-Run Fisher Relationship in Canada," *Canadian Journal of Economics*, v4b, 1124-1142.
- Crowder, W. J. and D. L. Hoffman (1996): "The Long-Run Relationship between Nominal Interest Rates and Inflation: The Fisher Equation Revisited," *Journal of Money, Credit and Banking*, v28, 102-118.
- Darby, M. R. (1975): "The Financial and Tax Effect of Monetary Policy on Interest Rates," *Economic Inquiry*, v13, 266-276.
- Davis, R. B. (1987): "Hypothesis Testing When a Nuisance Parameter Is Present Under the Alternative," *Biometrika*, v74, 33-43.
- Dickey, D. A. and W. A. Fuller (1979): "Distribution of the Estimators for Autoregressive Time Series With a Unit root," *Journal of American Statistical Association*, v74, 427-431.
- Diebold, F. X. and R. S. Mariano(1995): "Comparing Predictive Accuracy," *Journal of Business and Economic Statistics*, v13, 253-263

- Elliott, G., T. J. Rothenberg and J. H. Stock (1996): "Efficient Tests for an Autoregressive Unit Root," *Econometrica*, v64, 813-836.
- Enders, W. (2001): "Improved critical values for the Enders-Granger unit-root test," *Applied Economics Letters*, v8, 257-261.
- Enders, W. and C. W. J. Granger (1998): "Unit Root Tests and Asymmetric Adjustment with an Example Using the Term Structure of Interest Rates," *Journal of Business and Economic Statistics*, v16(3), 304-311.
- Enders, W. and P. Siklos (2001): "Cointegration and Threshold Adjustment," *Journal of Business and Economic Statistics*, v19(2), 166-176.
- Engle, R. F. and C. W. J. Granger (1987): "Cointegration and Error Correction Representation, Estimation, and Testing," *Econometrica*, v55, 251-276.
- Evans, M. D. D. and K. K. Lewis: (1995): "Do Expected Shifts in Inflation Affect Estimates of the Long-Run Fisher Relation?" *Journal of Finance*, v50, 225-253.
- Fama, E. F. (1975): "Short-Term Interest Rates as Predictors of Inflation," *The American Economic Review*, v65, 269-282.
- Fama, E. F. and M. R. Gibbons (1982): "Inflation, Real Returns and Capital Investment," *Journal of Monetary Economics*, v9, 297-323.
- Feldstein, M. (1976): "Inflation, Income Tax and Rate of Interest Rate: A Theoretical Analysis," *The American Economic Review*, v66, 809-820.
- Fisher, I. (1930): *The Theory of Interest*. The Macmillan Company.
- Hamilton, J. D. (1989): "A New Approach to the Economic Analysis of Nonstationary Time Series and Business Cycles," *Econometrica*, v57, 357-384.
- Hansen, B. E. (1999): "Testing Linearity," *Journal of Economic Surveys*, v13, 551-576.
- Hansen, B. E. and B. Seo (2002): "Testing for two-regime threshold cointegration in vector error correction models," *Journal of Econometrics*, v110, 293-318.
- Hess, G. D. and Iwata, S. (1997): "Asymmetric persistence in GDP? A deeper look at depth," *Journal of Monetary Economics*, v40, 535-554.
- Harvey, D. I., S. J. Leybourne, P. Newbold (1998): "Tests for Forecast Encompassing," *Journal of Business and Economic Statistics*, v16, 254-259.

- Johansen, S. (1988): "Statistical Analysis of Cointegrated Vectors," *Journal of Economic Dynamics and Control*, v12, 231-54.
- _____ (1991): "Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models," *Econometrica*, v6, 1551-1580.
- _____ (1994): "The Role of Constant and Linear Terms in Cointegration Analysis of Nonstationary Variables," *Econometric Reviews*, v13, 205-54.
- Johansen, S. and K. Juselius (1990): "Maximum Likelihood Estimation and Inferences on Cointegrating with Application to the Demand for Money," *Oxford Bulletin of Economics and Statistics*, v52, 169-209.
- Kesriyeli, M. et al (2004): "Nonlinearity in Interest Rate Reaction Functions for the US, the UK and Germany," *working paper*.
- Lo, M. C. and E. Zivot (2000): "Threshold Cointegration and Nonlinear Adjustment to the Law of One Price," *Macroeconomic Dynamics*, v5, 533-576.
- Malliaropulos, D. (2000): "A Note on Nonstationarity, Structural Breaks, and the Fisher Effect," *Journal of Banking and Finance*, v24, 695-707.
- Mishkin, F. S. (1992): "Is the Fisher Effect for Real?" *Journal of Monetary Economics*, v30, 195-215.
- Neftci, S. N. (1984): "Are Economic Time Series Asymmetric over the Business Cycle?" *Journal of Political Economy*, v92, 307-328.
- Nelson, C. R. (1972): "The Prediction Performance of the FRB-MIT-PENN Model of the US Economy," *American Economic Review*, v62, 902-917.
- Ng, S. and Perron, P. (1995): "Unit Root Tests ARMA Models with Data Dependent Methods for the Selection of the Truncation Lag," *Journal of American Statistics Association*, v90, 268-291.
- _____ (2001): "LAG Length Selection and the Construction of Unit Root Tests with Good Size and Power," *Econometrica*, v69, 1519-1554.
- Newey, W. K. and K. D. West (1987): "A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix," *Econometrica*, v55, 703-708.
- Perron, P. (1989): "The Great Crash, the Oil Price Shock, and the Unit Root Hypothesis," *Econometrica*, v57, 1361-1401.

- Petrucelli, J. D. and N. Davis (1986): "A Portmanteau Test for Self-Exciting Threshold Autoregressive-Type Nonlinearity in Time Series," *Biometrika*, v73, 687-694.
- Phillips, P. C. B. (1987): "Time Series Regression with a Unit Root," *Econometrica*, v55, 277-301.
- Phillips, PCB and P. Perron (1988): "Testing for a Unit Root in Time Series Regressions", *Biometrika*, V75, 335-346.
- Phillips, PCB and S. Ouliaris (1990): "Asymptotic properties of residual based tests for cointegration", *Econometrica*, v58, 165-193.
- Quandt, R. E. (1958): "The Estimation of the Parameters of a Linear Regression System Obeying Two Separate Regimes," *Journal of American Statistical Association*, v53, 873-880.
- Rose, A. K. (1988), "Is the Real Interest Rate Stable." *Journal of Finance*, v43, 1095-1112.
- Sidrauski, M. (1967): "Inflation and Economic Growth," *Journal of Political Economy*, v75, 796-810.
- Stock, J. H. and M. W. Watson (1993): "A Simple Estimator of Cointegrating Vectors in Higher Order Integrated Systems," *Econometrica*, v61, 783-820.
- Stockman, A. C. (1981): "Anticipated Inflation and the Capital Stock in a Cash-in-Advance Economy," *Journal of Monetary Economics*, v8, 387-393.
- Tobin, J. (1965): "Money and Economic Growth," *Econometrica*, v33, 671-684.
- Tsay, R. (1989): "Testing and Modeling Threshold Autoregressive Processes," *Journal of American Statistical Association*, v84, 231-240.
- Tsay, R. (1998): "Testing and Modeling Multivariate Threshold Models," *Journal of American Statistical Association*, v93, 1188-1202.
- Wang, P., and C. Yip (1992): "Alternative Approaches to Money and Growth," *Journal of Money, Credit and Banking*, v24, 553-563.
- Weidmann, J. (1997): "New Hope for the Fisher Effect? A reexamination Using Threshold Cointegration," University of Bonn, working paper.
- Zivot, E. and D. Andrews (1992): "Further Evidence on the Great Crash, the Oil Price Shock, and the Unit Root Hypothesis," *Journal of Business and Economic Statistics*, v10, 251-270.

ACKNOWLEDGEMENTS

I would like to thank the members of my committee: Dr. Helle Bunzel, Dr. Barry Falk, Dr. Wayne Fuller, Dr. Peter Orazem and Dr. John Shroeter for their time, valuable comments and instructions in the whole process. Special thanks should be given to Dr. Barry Falk, my major professor, for his guidance and encouragement.

I dedicate this dissertation to my parents, Quanlin Xu and Weifeng Liu, who have formed me into the person I am today. It is their constant efforts, encouragement and hard work that have made achieving the goal of obtaining a Ph.D. possible. My father passed away one year after I entered the PH.D program, yet he has always been my source of courage and inspiration.